A Data Analytics Paradigm for the Construction, Selection, and Evaluation of Mortality Models

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Acknowledgments

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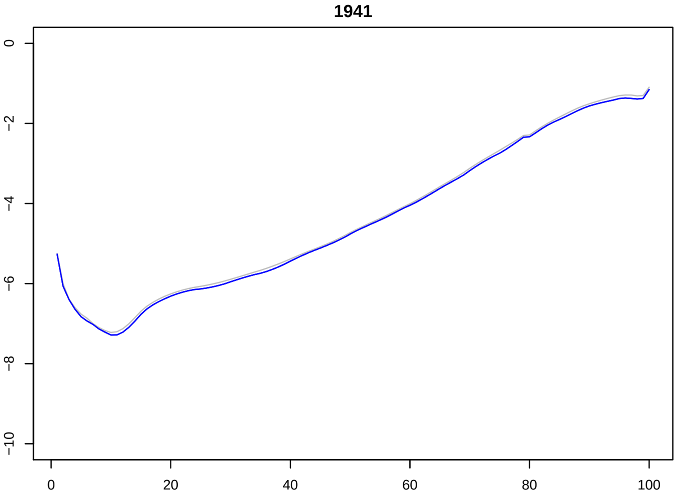
Agenda

- Review of Mortality Models
- Review of Data Analytics Techniques
- Construction via Regularisation
- Selection via Cross Validation
- Evaluation via Out-of-Sample Forecasting
- Summary

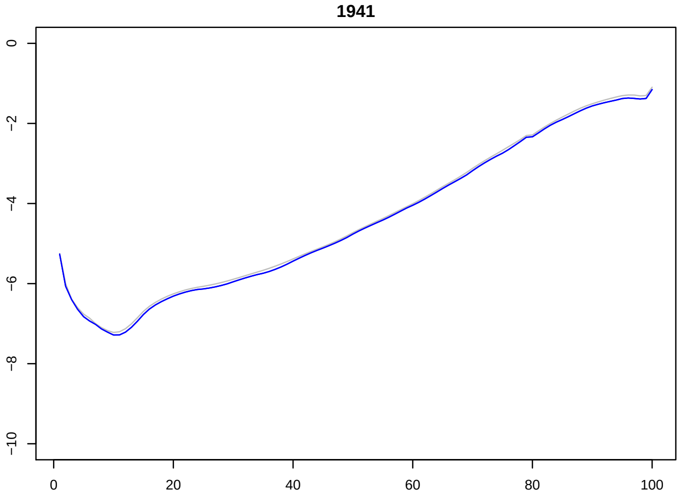
A Data Analytics Paradigm for the Construction, Selection, and Evaluation of **Mortality Models**

Mortality Models: Motivation



$$\ln(\mu_{x,t}) =$$

Mortality Models: Motivation



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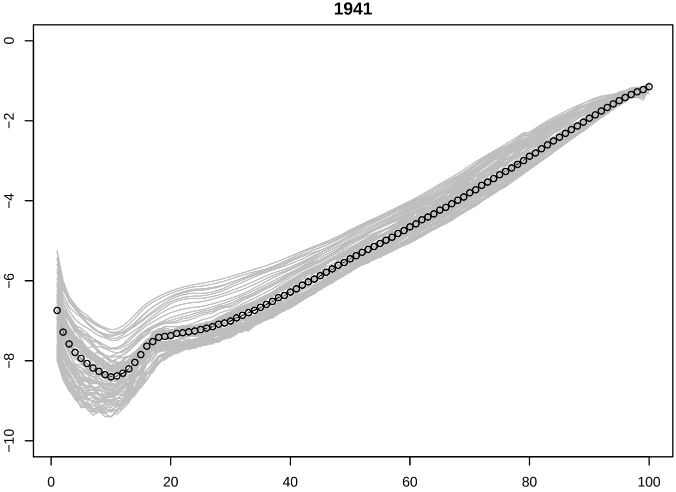
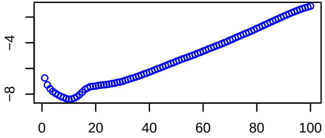
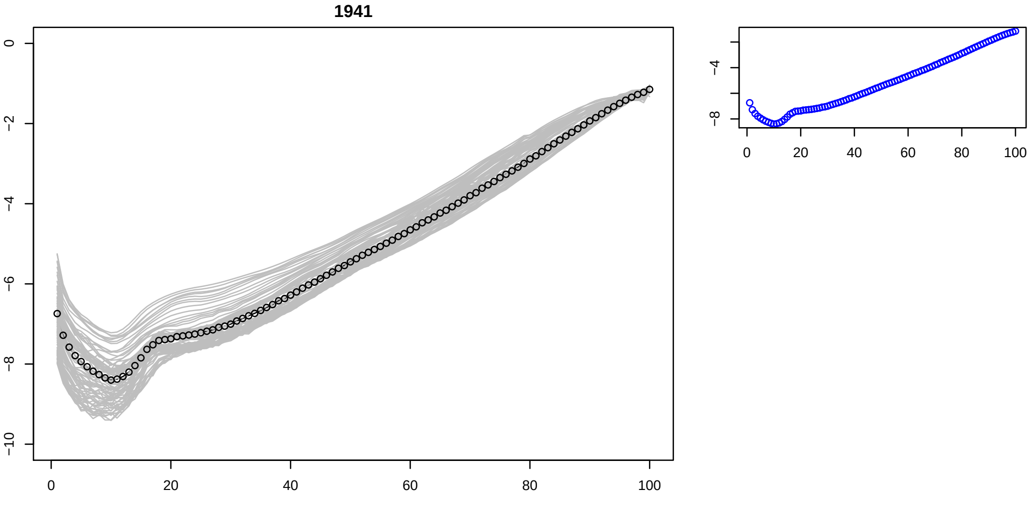
Mortality Models: Constructing Mortality Models





$$\ln(\mu_{x,t}) = \alpha_x$$

Mortality Models: Constructing Mortality Models



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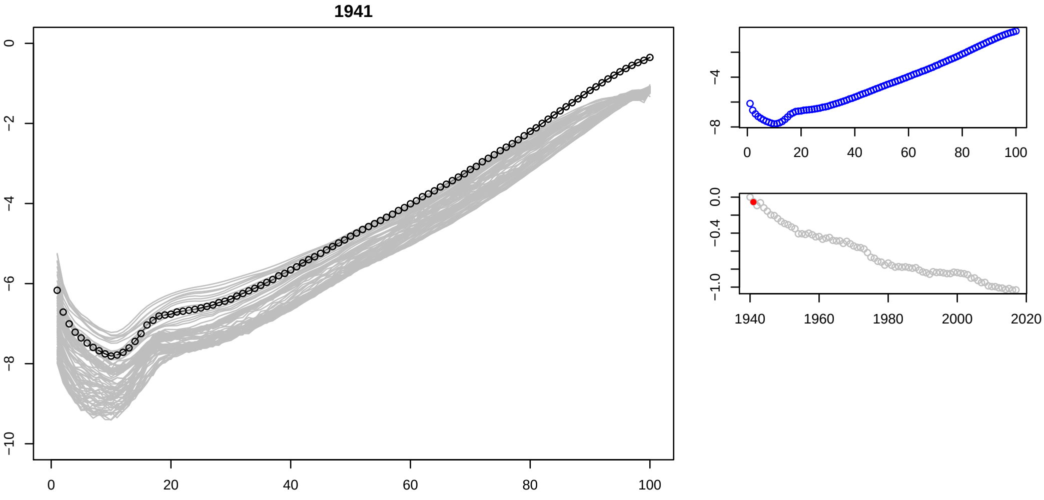
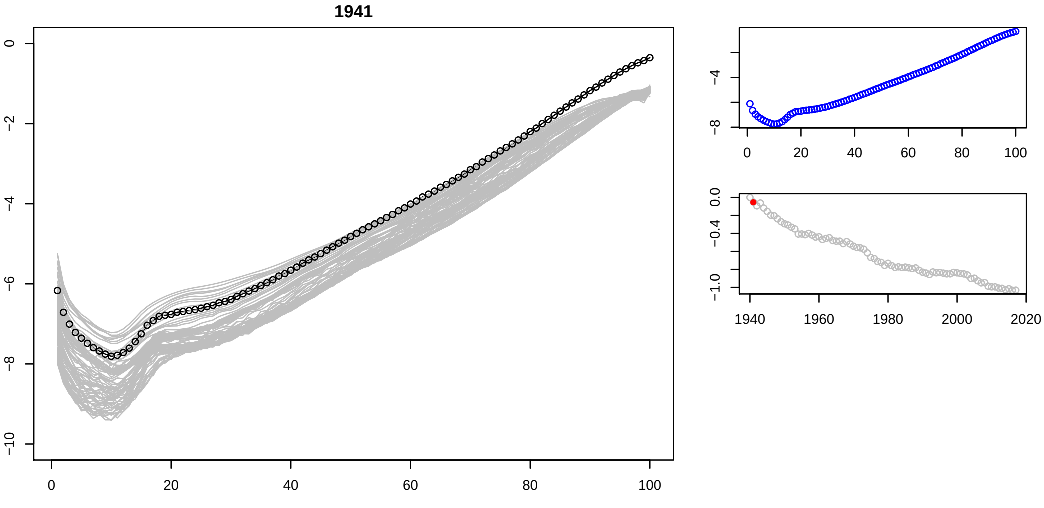


$$\ln(\mu_{x,t}) = \alpha_x + \kappa_t^{(1)}$$

Mortality Models: Constructing Mortality Models



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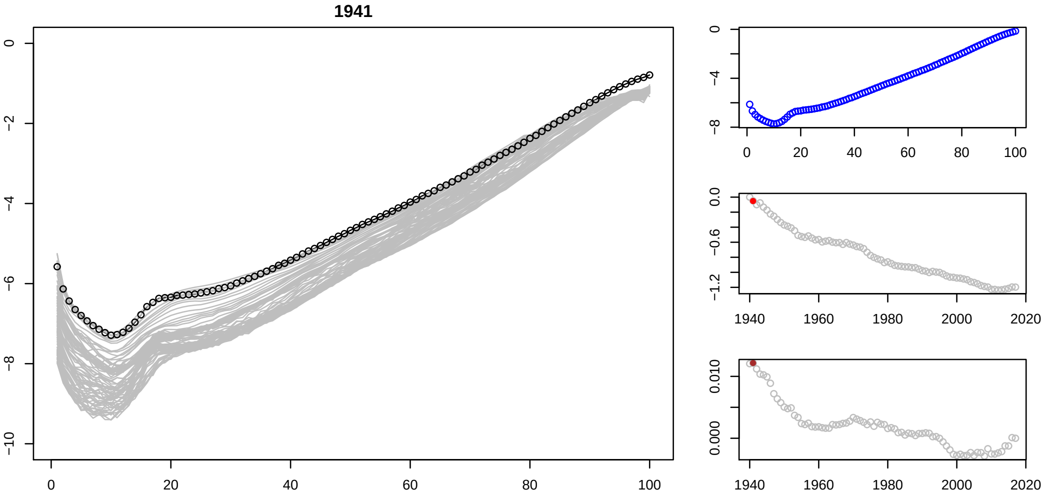
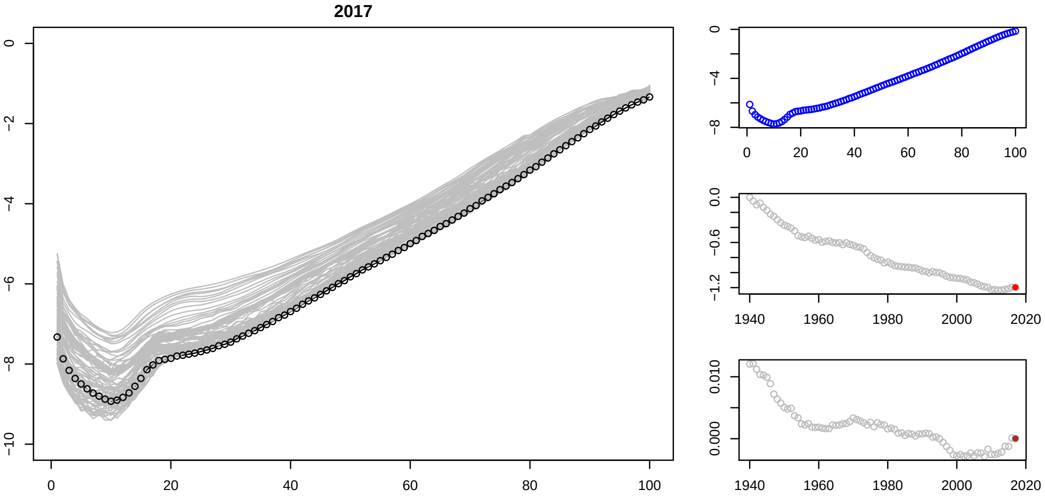


$$\ln(\mu_{x,t}) = \alpha_x + \kappa_t^{(1)} + (\bar{x} - x)\kappa_t^{(2)}$$

Mortality Models: Constructing Mortality Models



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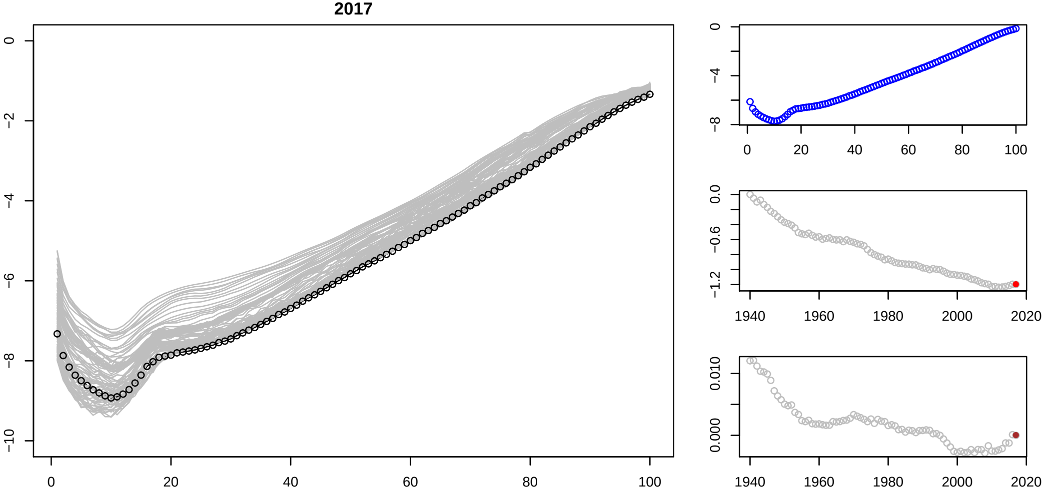


$$\ln(\mu_{x,t}) = \alpha_x + \kappa_t^{(1)} + (\bar{x} - x)\kappa_t^{(2)} + \gamma_c$$

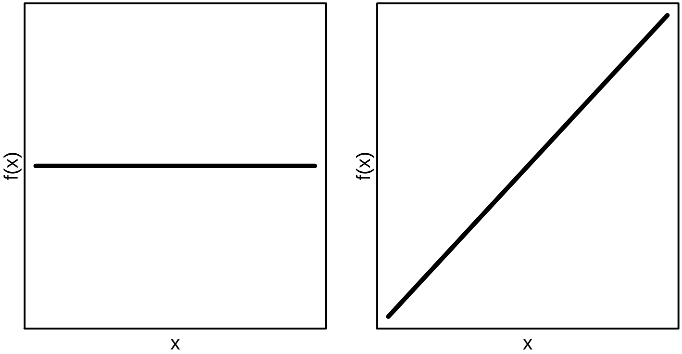
Mortality Models: Generalised Age-Period-Cohort Models

$$ln(\mu_{x,t}) = \alpha_x + \sum_{i=1}^{N} f^{(i)}(x)\kappa_t^{(i)} + \gamma_c$$

Name	Form	Parameters
LC	$\alpha_x + \beta_x^{(1)}\kappa_t^{(1)}$	$2n_a + n_y$
LC2	$\alpha_x + \beta_x^{(1)}\kappa_t^{(1)} + \beta_x^{(2)}\kappa_t^{(2)}$	$3n_a + 2n_y$
RH	$\alpha_x + \beta_x^{(1)}\kappa_t^{(1)} + \beta_x^{(0)}\gamma_c$	$3n_a + n_y + n_c$
APC	$\alpha_x + \kappa_t^{(1)} + \gamma_c$	$n_a + n_y + n_c$
CBD	$\kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)}$	$2n_y$
M7	$\kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)} + ((x - \bar{x})^2 - \sigma_x^2)\kappa_t^{(3)} + \gamma_c$	$3n_y + n_c$
sPLAT	$\alpha_x + \kappa_t^{(1)} + (\bar{x} - x)\kappa_t^{(2)} + \gamma_c$	$n_a + 2n_y + n_c$
cPLAT	$\alpha_x + \kappa_t^{(1)} + (\bar{x} - x)\kappa_t^{(2)} + (\bar{x} - x)^+\kappa_t^{(3)} + \gamma_c$	$n_a + 3n_y + n_c$

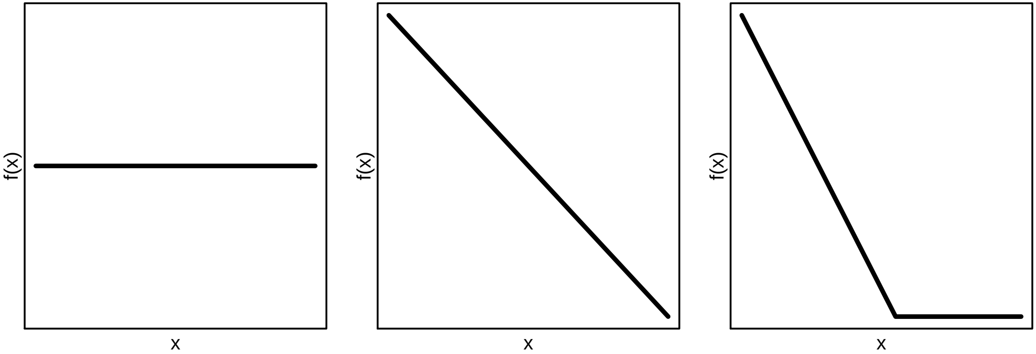
Mortality Models: CBD Model

$$\eta_{x,t} = \kappa_t^{(1)} + (x - \bar{x})\,\kappa_t^{(2)}$$



Mortality Models: cPLAT Model

$$\eta_{x,t} = \alpha_x + \kappa_t^{(1)} + (\bar{x} - x)\,\kappa_t^{(2)} + (\bar{x} - x)^+\,\kappa_t^{(3)} + \gamma_{t-x}$$



A **Data Analytics** Paradigm for the Construction, Comparison, and Selection of Mortality Models

Data Analytics: Predictive Modelling

Objective: Predict response using linear combination of predictors,

$$y_i = \beta_0 + \sum_{j=1}^{p} x_{ij}\beta_j + \epsilon_j, \tag{1}$$

where β_0 and $\beta = (\beta_1, \beta_2, ..., \beta_p)$ are unknown parameters.

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Typically, parameters are estimated by minimising OLS,

$$\frac{1}{N}\sum_{i=1}^{N}(y_i - \beta_0 - \sum_{j=1}^{p} x_{ij}\beta_j)^2. \tag{2}$$

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Problem: *Interpretation* and *prediction accuracy*.

Data Analytics: Regularisation

Solution: Minimise

$$\frac{1}{N}\sum_{i=1}^{N}(y_i - \beta_0 - \sum_{j=1}^{p} x_{ij}\beta_j)^2, \tag{3}$$

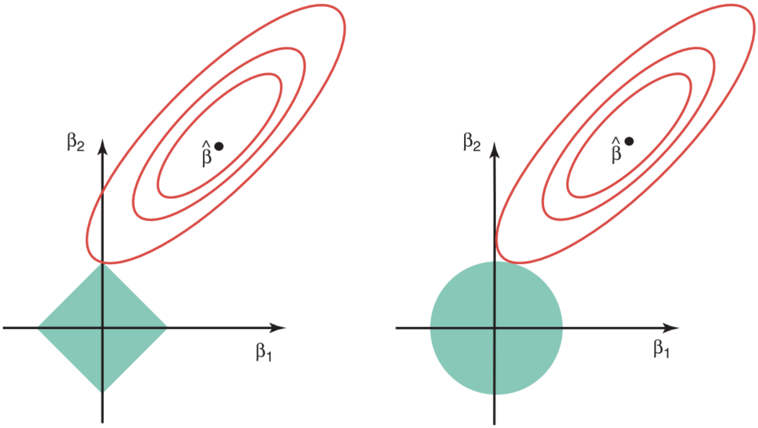
subject to ℓ_2-norm constraint (ridge regression),

$$\sum_{j=1}^{p}\beta_j^2 \leq t, \tag{4}$$

or ℓ_1-norm constraint (lasso),

$$\sum_{j=1}^{p}|\beta_j| \leq t. \tag{5}$$

Data Analytics: Lasso and Ridge Constraints



James et al. (2014)

Data Analytics: Group Lasso

Consider the linear predictor structure,

$$\boldsymbol{y} = \mathbf{X}\beta = \sum_{j=1}^{J} \mathbf{X}_j \boldsymbol{\beta}_j, \tag{6}$$

where $\boldsymbol{\beta}_j$ is a vector of coefficients for group j.

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The group lasso minimises the objective function,

$$Q(\boldsymbol{\beta}|\mathbf{X}, \boldsymbol{\eta}) = L(\boldsymbol{\beta}|\mathbf{X}, \boldsymbol{\eta}) + \sum p_\lambda(\|\boldsymbol{\beta_j}\|), \tag{7}$$

where $p_\lambda(\cdot)$ is a penalty applied to the ℓ_2-norm of each group.

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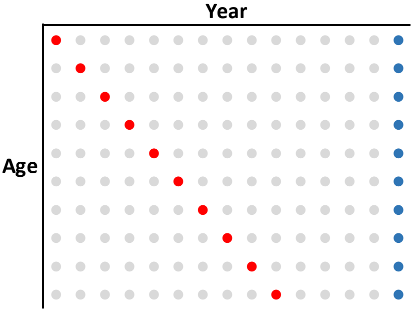
If group j is selected, $\beta_{jk} \neq 0$ for all k; else $\beta_{jk} = 0$ for all k.

A Data Analytics Paradigm for the Construction, Comparison, and Selection of Mortality Models

Construction: Data



Data Source: Human Mortality Database (30 Countries, 2 Genders)

Years: 1960 - 2015

Ages: 20 - 89

Construction: “Formalised” Model-Building Framework

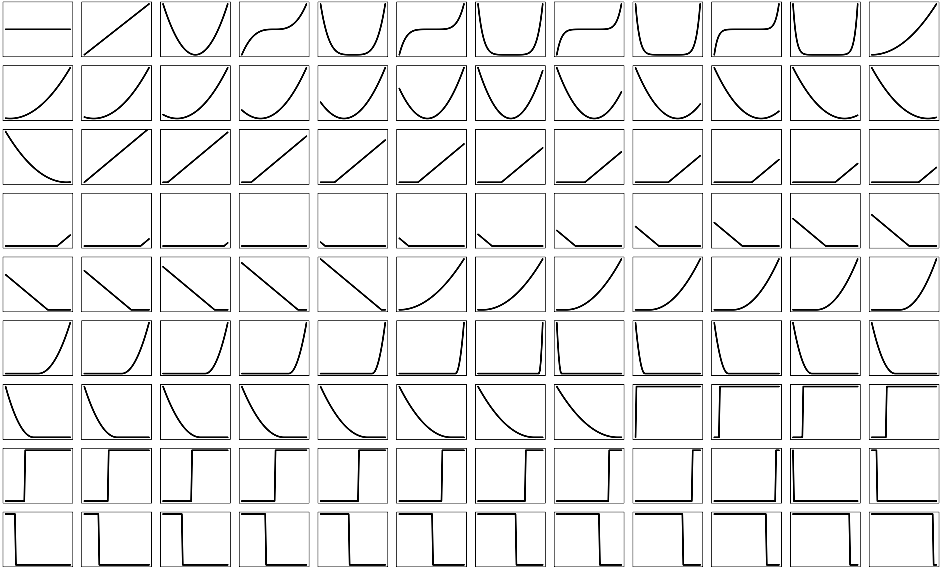
We start with a **huge** model,

$$\ln(\mu_{x,t}) = \alpha_x + \sum_{i=1}^{B} f^{(i)}(x)\kappa_t^{(i)} + \gamma_{t-x},$$

where the suite of basis functions $(f^{(i)}(x))$ included are:

$$f^{(i)}(x) = \begin{cases} 1, & \text{Unit} \\ (x-\bar{x})^n & \text{Polynomial} \\ (x-n)^+ & \text{Call} \\ (n-x)^+ & \text{Put} \\ 1_{x<n} & \text{Below} \\ 1_{x>n} & \text{Above} \end{cases} . \tag{8}$$

Construction: Our Model



Construction: GLM Representation

$$\eta_{x,t} = \ln(\mu_{x,t}) = \alpha_x + \sum_{i=1}^{B} f^{(i)}(x)\kappa_t^{(i)} + \gamma_c,$$

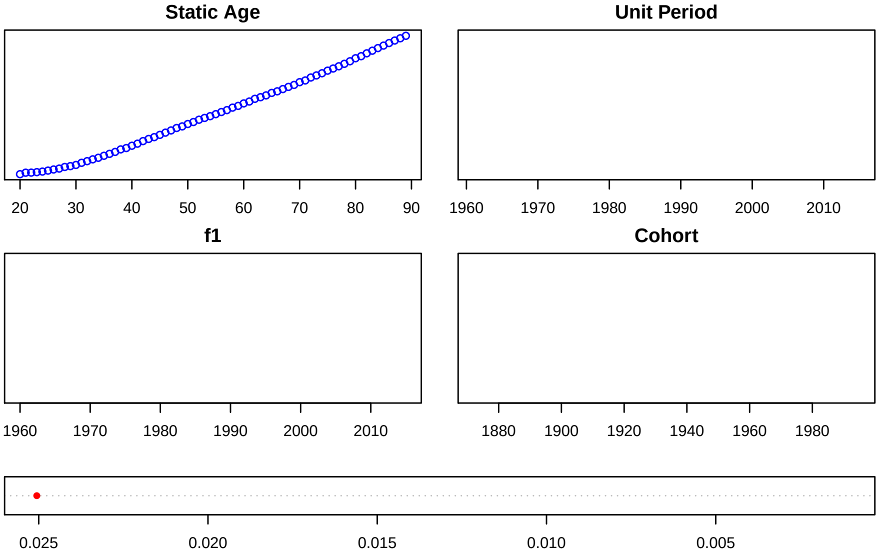
can be expressed as a GLM,

$$\eta = \mathbf{X}\beta = \sum_{j=0}^{B+1} \mathbf{X}_j \boldsymbol{\beta}_j, \qquad \mathbf{X} = [\mathbf{X_0} : \mathbf{X_1} : \mathbf{X_2} : \ldots : \mathbf{X_B} : \mathbf{X_{B+1}}],$$

where,

$$\boldsymbol{\beta} = \{\boldsymbol{\beta_i}\}_{i=0}^{B+1}, \quad \boldsymbol{\beta_0} = \{\alpha_x\}_{x=1}^{n_x}, \quad \boldsymbol{\beta_i} = \{\kappa_t^{(i)}\}_{t=1}^{n_t}, \quad \boldsymbol{\beta_{B+1}} = \{\gamma_c\}_{c=1}^{n_c}.$$

Construction: Estimated Parameters



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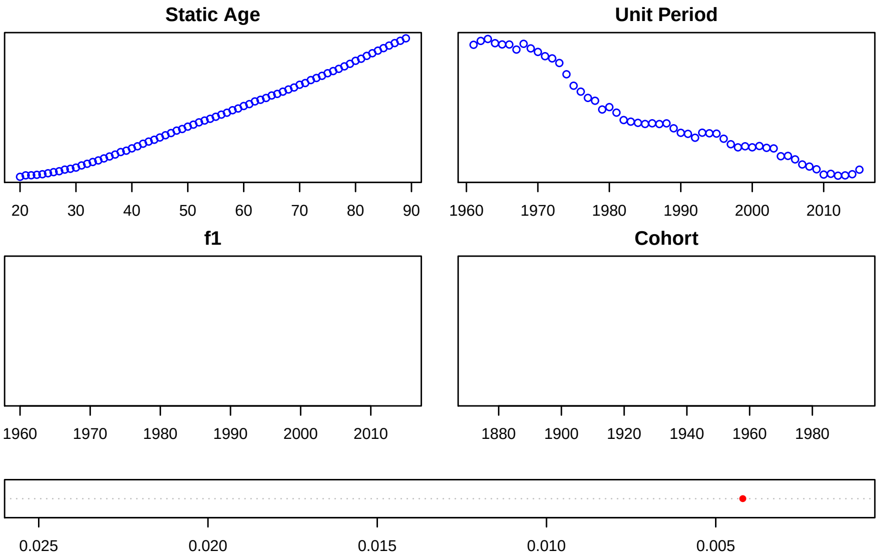

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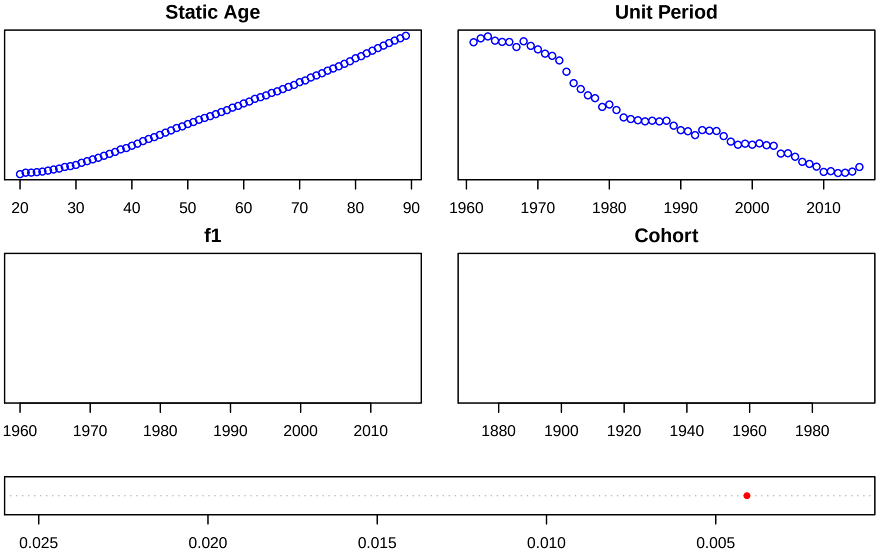
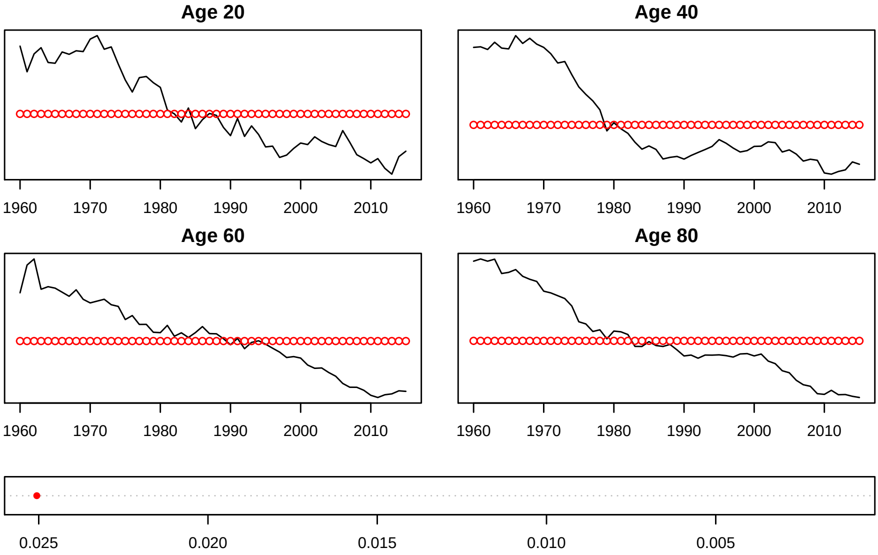


Construction: Fitted Values



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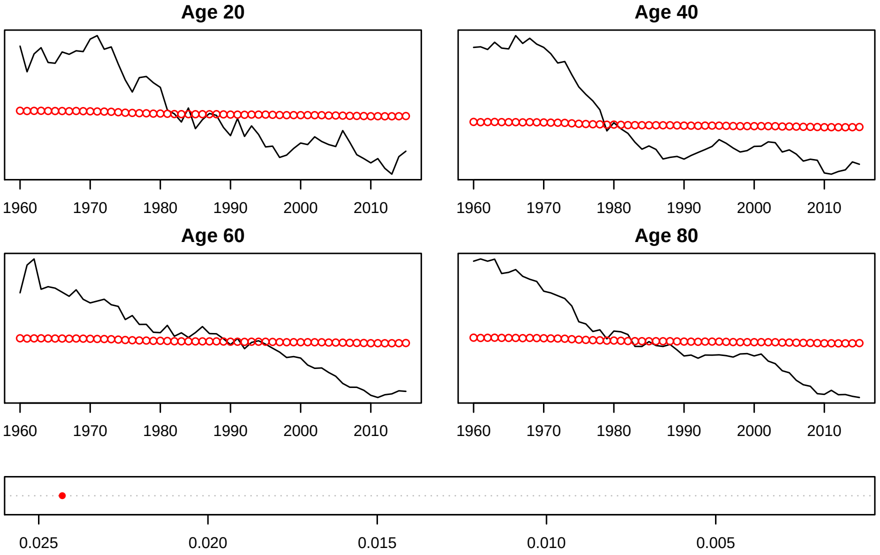

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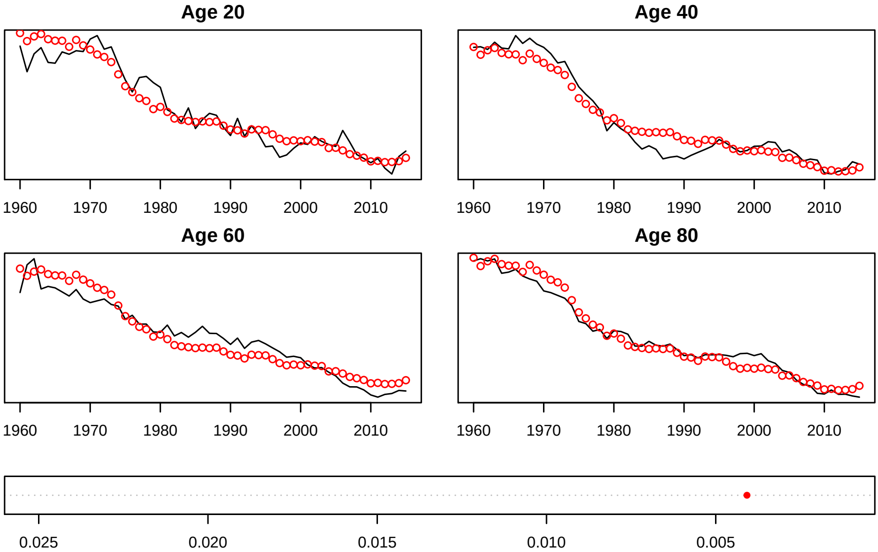
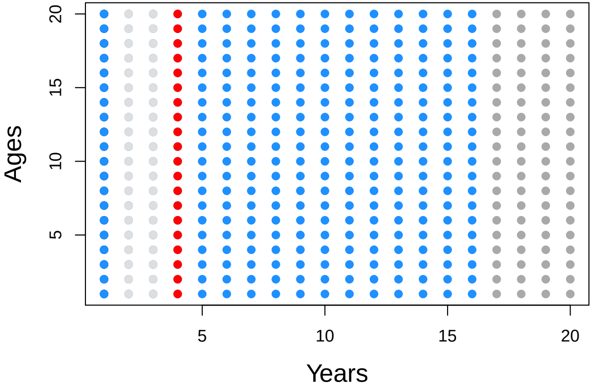


Selection and Evaluation: Cross Validation Framework

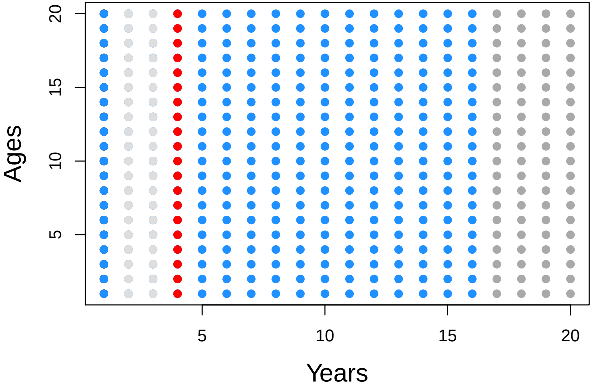


Training/Test Set: 1960 - 1990

Validation Set: 1991 - 2015

Test Set Width: Forecasting Horizon

Selection and Evaluation: Cross Validation Framework



Training/Test Set: 1960 - 1990

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Test Set Width: Forecasting Horizon

Selection Results: Optimal Models

Data	1 Year Horizon	Parameters
USA (F)	$\alpha_x + \kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)} + \gamma_c$	262
AUS (M)	$\alpha_x + \kappa_t^{(1)} + (x - \bar{x})^2\kappa_t^{(2)} + (x - 25)^+\kappa_t^{(3)} + (25 - x)^+\kappa_t^{(4)} + \gamma_c$	344
UK (M)	$\alpha_x + \kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)} + (x - \bar{x})^2\kappa_t^{(3)} + \gamma_c$	303
CAN (F)	$\alpha_x + \kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)} + (25 - x)^+\kappa_t^{(3)} + \gamma_c$	303

Data	10 Year Horizon	Parameters
USA (F)	$\alpha_x + \kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)} + \gamma_c$	262
AUS (M)	$\alpha_x + \kappa_t^{(1)} + (x - \bar{x})^2\kappa_t^{(2)} + \gamma_c$	262
UK (M)	$\alpha_x + \kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)} + (x - \bar{x})^2\kappa_t^{(3)} + \gamma_c$	303
CAN (F)	$\alpha_x + \kappa_t^{(1)}$	111

Evaluating Results: By Forecasting Horizon

	1 Year	Rank 1	Rank 2	Rank 3	Rank 4	Rank 5	Rank 6	Rank 7	Rank 8	Rank 9
1	GL	27	10	16	5	1	0	1	0	0
2	LC2	21	14	6	7	7	4	1	0	0
3	cPLAT	4	13	3	6	6	17	9	2	0
4	LC	3	6	6	9	3	10	10	8	5
6	APC	1	10	12	15	14	4	2	2	0
8	sPLAT	0	1	7	7	15	12	13	5	0

	10 Years	Rank 1	Rank 2	Rank 3	Rank 4	Rank 5	Rank 6	Rank 7	Rank 8	Rank 9
1	GL	15	17	9	11	5	3	0	0	0
2	APC	15	11	13	15	2	3	1	0	0
3	LC	13	10	11	15	10	1	0	0	0
4	LC2	7	15	22	11	4	1	0	0	0
7	sPLAT	0	0	1	0	4	19	21	10	5
8	cPLAT	0	0	0	1	3	5	25	22	4

Evaluating Results: By Population Size

Size (m)	LC	LC2	CBD	M7	APC	RH	sPLAT	cPLAT	GL
0.00-2.50	5.1	1.2	7.2	7.4	3.4	5.3	6.5	5.9	3.0
2.50-3.75	3.8	2.3	7.7	8.2	3.6	4.5	6.5	5.9	2.5
3.75-5.00	5.1	2.4	8.3	7.3	3.5	5.2	5.8	5.7	1.7
5.00-10.00	4.1	2.6	9.0	7.4	3.5	5.3	5.8	5.1	2.0
10.00-25.00	6.6	3.4	8.7	5.7	4.9	6.0	4.5	3.4	1.8
$>$25.00	7.0	3.9	8.8	5.9	4.8	6.7	4.2	1.9	1.8

Key Takeaways

- Complex models perform well for short-term forecasting;
- Simpler models perform well for long-term forecasting;
- Complex models perform well on large populations;
- Complex models perform poorly on small populations;
- Simpler models perform relatively consistently across population size;
- Complex models produce very volatile outputs;
- Small, less developed populations do not exhibit strong cohort effects.

No pre-defined model will perform well for all contexts.

Future work and limitations

- **Estimation**: Gaussian assumption
  - Extension to a Poisson setting
- **Selection**: Based exclusively on MSE (point estimates)
  - Interval based-measures, life-span disparity
- **Construction**: Focus on single population mortality rates
  - Multiple populations, improvement rates
- Integration of the new techinques into the R Package **StMoMo**

Questions?

Email: a.villegas@unsw.edu.au