Network-Based Optimal Control of Pollution Growth

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- Spatial economic and ecological patterns generated by **transboundary pollution** on **network**.
- Pollution, as a **negative externality**, originates locally, spreads to distant areas and is included into economic decisions.
- **Central planner** that controls **investments** (in green and brown production) and **depollution efforts** aiming to **maximise the social welfare**.
- Exension to network of results on continous time-space e.g. Fabbri, Boucekkine, Gozzi, Federico [BFFG21],[BFFG19], [BFFG22]; Similar formulation in De Frutos et al. [DFMH19],[DFLPMH21].

In each site $i \in V = \{1, \dots, n\}$ it holds:

$$Y_i(t) = a_i^I(t) I_i(t) + a_i^R(t) R_i(t),$$

and

$$C_i(t) + I_i(t) + R_i(t) + B_i(t) = Y_i(t),$$

for

- $Y_i(t)$ **production** at time t and location i;
- $I_i(t)$ **non-renewable** investment at time t and location i;
- $R_i(t)$ **renewable** investment at time t and location i;
- $a_i^I(t)$, $a_i^R(t) \geq 1$ productivity factors;
- $C_i(t)$ **consumption** at time t and location i;
- $B_i(t)$ **abatement** level at time t and location i.

Pollution evolves according to:

$$\begin{cases} \frac{d}{dt}P_i(t) = \underbrace{\sum_{j=1}^{n} L_{ij}P_j(t)}_{\text{inflow}} - \underbrace{\sum_{j=1}^{n} L_{ji}P_i(t)}_{\text{outflow}} - \underbrace{\delta_i P_i(t)}_{\text{decay}} + \underbrace{I_i(t) + \varepsilon_i R_i(t)}_{\text{pollution}} - \underbrace{\varphi_i(B_i(t))^{\theta}}_{\text{depollution}}, \\ P_i(0) = p_i \geq 0. \end{cases}$$

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Consider a social planner aiming at maximizing the social welfare

$$J(p,(I,R,B)) := \int_0^{+\infty} e^{-\rho t}\left(\sum_{i=1}^n \left(\frac{C_i(t)^{1-\gamma}}{1-\gamma} - \omega_i P_i(t) - f_i(R_i(t))\right)\right)dt, \tag{1}$$

where

- ω_i local **environmental awareness**
- $f_i(R_i(t))$ convex **maintenance and operational cost** related to renewable investments

Theorem

Under suitable assumptions

- *the optimal control problem admits a* ***unique solution*** (I^*, R^*, B^*);
- *Found explicit solution for* $f_i(R_i) = \lambda R_i$
- *the optimal spatial pollution density* $P(t)$ *converges as* $t \to \infty$ *to the* ***long-run*** *pollution profile* P_∞, *unique solution to the following ODE:*

$$(L - \delta)P_\infty + I^*(t) + \varepsilon R^*(t) - \varphi B^*(t)^\theta = 0.$$

where

- $L = (L_{i,j})_{i,j=1,\dots,n}$ *diffusive linear operator,*
- ε *pollution intensity factor associated to the renewable investment,*
- φ *efficiency of abatement,*
- θ *return to scale of abatement.*

Numerical Example: Spatial discrepancy in input productivity

$n = 20$ nodes

$$L = \begin{cases} \frac{1}{n} & i \neq j \\ -\frac{n-1}{n} & i = j \end{cases}$$

$\rho = 0.03$

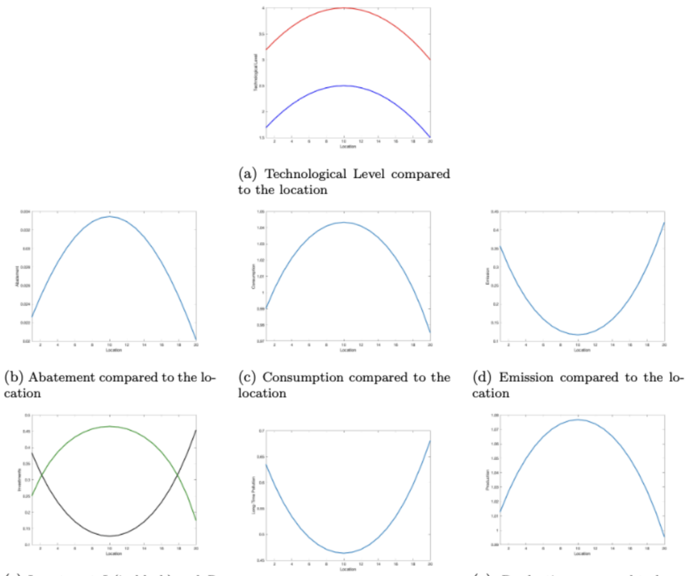
$\phi = 0.11$

$\theta = 0.2$

$\omega_i = 1$

$f_i(R_i) = \lambda R_i^2 \forall i$

$\lambda = 1$



(a) Technological Level compared to the location

(b) Abatement compared to the location

(c) Consumption compared to the location

(d) Emission compared to the location

(e) Investment I (in black) and R (in green) compared to location

(f) Pollution compared to location

(g) Production compared to location

Numerical Example: Impact of the cost parameter λ

n = 20 nodes

$$L = \begin{cases} \frac{1}{n} & i \neq j \\ -\frac{n-1}{n} & i = j \end{cases}$$

$\rho = 0.03$

$\phi = 0.11$

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$\omega_i = 1$

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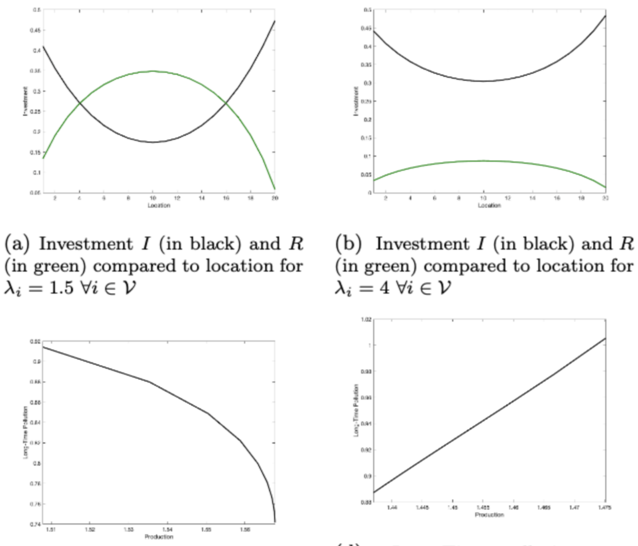

(a) Investment I (in black) and R (in green) compared to location for $\lambda_i = 1.5\ \forall i \in \mathcal{V}$

(b) Investment I (in black) and R (in green) compared to location for $\lambda_i = 4\ \forall i \in \mathcal{V}$

(c) Long-Time pollution compared to Production for $\lambda_i = 1.5\ \forall i \in \mathcal{V}$

(d) Long-Time pollution compared to Production for $\lambda_i = 4\ \forall i \in \mathcal{V}$

References

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