

Non-crossing neural network quantile regression estimation for driving data with telematics

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Risk evaluation

• Evaluation of heavy tailed distributions is a crucial part of risk assessment.

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Risk evaluation

- Evaluation of heavy tailed distributions is a crucial part of risk assessment.
- When datasets present **long conditional tails** on their response variables, algorithms based on **Quantile Regression** have been widely used to assess extreme quantile behaviors.

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Value at Risk (VaR) and Conditional Tail Expectation (CTE)

Definition

$$\mathsf{VaR}_q(Y) = \inf\{y \in \mathbb{R} | \mathsf{F}_Y(y) > q\} = \mathsf{F}_Y^{-1}(1-q)$$

where F_Y is the distribution function of the random continuous variable Y.

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Definition

$$\mathsf{CTE}_q(Y) = \mathbb{E}[Y|Y \geq \mathsf{VaR}_q(Y)]$$

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Expectation (CTE)



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Value	at Diale (d Canditi		

Expectation (CTE)



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Elicitat	oility of	CTE			

• The definition of elicitability can be reduced into the **existence of a scoring function** that is strictly consistent (Gneiting (2011)).

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Elicitab	ility of	CTE			

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- Acerbi and Szekely (2014) found a consistent scoring function but did not opened the discussion of elicitability.

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Elicitat	oility of	CTE			

- The definition of elicitability can be reduced into the **existence of a scoring function** that is strictly consistent (Gneiting (2011)).
- Acerbi and Szekely (2014) found a consistent scoring function but did not opened the discussion of elicitability. Afterwards, Fissler and Ziegel (2016) prove that **CTE alone it is not elicitable, but the pair (VaR, CTE) is**.

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Scoring functions

Scoring Function - VaR (Koenker and Bassett Jr (1978))

$$\rho_q(r_1, y) = (q - \mathbb{1}_{\{y-r_1 < 0\}})(y - r_1)$$

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Scoring functions

Scoring Function - VaR (Koenker and Bassett Jr (1978))

$$\rho_q(r_1, y) = (q - \mathbb{1}_{\{y - r_1 < 0\}})(y - r_1)$$

Scoring Function - CTE (Fissler and Ziegel (2016))

$$S_q(r_1, r_2, y) = \mathbb{1}_{\{y > r_1\}} \big(-G_1(r_1) + G_1(y) - G_2(r_2)(r_1 - y) \big) + (1 - q) \big(G_1(r_1) - G_2(r_2)(r_2 - r_1) + \mathcal{G}_2(r_2) \big)$$

with G_1 being an increasing function, \mathcal{G}_2 an increasing and concave function and $\mathcal{G}_2' = G_2$

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• Acerbi and Szekely (2014) realized a non-crossing problem between the VaR and the CTE

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• Acerbi and Szekely (2014) realized a non-crossing problem between the VaR and the CTE, namely $VaR_q(y_i) \leq CTE_q(y_i)$.

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- Acerbi and Szekely (2014) realized a non-crossing problem between the VaR and the CTE, namely $VaR_q(y_i) \leq CTE_q(y_i)$.
- But what about several quantile levels?

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• The problem of crossing quantiles' line of research was created by He (1997) and Yu et al. (2003)

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Non-cro	ssing al	gorithm	s 2		

• The problem of crossing quantiles' line of research was created by He (1997) and Yu et al. (2003), namely $VaR_{q_0}(y_i) \leq VaR_{q_1}(y_i)$

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- The problem of crossing quantiles' line of research was created by He (1997) and Yu et al. (2003), namely $VaR_{q_0}(y_i) \leq VaR_{q_1}(y_i)$
- Recent advances use neural networks (see Cannon (2018) and Moon et al. (2021))

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- The problem of crossing quantiles' line of research was created by He (1997) and Yu et al. (2003), namely $VaR_{q_0}(y_i) \leq VaR_{q_1}(y_i)$
- Recent advances use neural networks (see Cannon (2018) and Moon et al. (2021))
- Vidal-Llana et al. (2022) presented an approach to a multiple quantile levels for VaR and CTE estimation with non-crossing conditions

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• Showcase the problematic of crossing quantiles across VaRs and between a VaR and its CTE under a telematics context

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- Showcase the problematic of crossing quantiles across VaRs and between a VaR and its CTE under a telematics context
- Compare a classical approach to several quantile levels against a methodology that assures non-crossing conditions

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Dataset

Telematic information from year 2015 of 9,614 drivers from a Spanish insurance company



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Crossing example



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Crossings on a Two Step approach

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Crossings on a Two Step approach

q_i - q_{i+1}	0.9 - 0.925	0.925 - 0.95	0.95 - 0-975	0.975 - 0.99
$VaR_{q_i} > VaR_{q_{i+1}}$	3 (0%)	1 (0%)	2 (0%)	2 (0%)
$CTE_{q_i} > CTE_{q_{i+1}}$	0 (0%)	541 (6%)	1,560 (16%)	176 (2%)

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Murphy Diagrams: CTE comparison



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Loss improvement



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Conclus	sions				

• Inside an insurance company pricing scheme, **crossing predictions become unfeasible estimations**, thus the usefulness of non-crossing algorithms

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Conclusi	ons				

- Inside an insurance company pricing scheme, **crossing predictions become unfeasible estimations**, thus the usefulness of non-crossing algorithms
- For financial practitioners, and after Basel III recommendations, non-crossing predictions help assess bank reserves in a more consistent way

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Additional results I



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Additional results II



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GitHub Repository

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Non-Crossing Dua	Non-Crossing Dual Neural Network					
This is a repository in regards of the article "Non-Crossing Dual Neural Network: Joint Value at Risk and Conditional Tail Expectation estimations with Non-Crossing Conditions" (Working Paper).						

GitHub NCDNN Repository

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Referen	ces				

- Acerbi, C. and Szekely, B. (2014). Back-testing expected shortfall. *Risk*, 27(11):76–81.
- Cannon, A. J. (2018). Non-crossing nonlinear regression quantiles by monotone composite quantile regression neural network, with application to rainfall extremes. *Stochastic Environmental Research and Risk Assessment*, 32(11):3207–3225.
- Fissler, T. and Ziegel, J. F. (2016). Higher order elicitability and Osband's principle. *The Annals of Statistics*, 44(4):1680–1707.
- Gneiting, T. (2011). Making and evaluating point forecasts. *Journal of the American Statistical Association*, 106(494):746–762.

He, X. (1997). Quantile curves without crossing. *The American Statistician*, 51(2):186–192.

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List of References

- Koenker, R. and Bassett Jr, G. (1978). Regression quantiles. *Econometrica:* Journal of the Econometric Society, 46(1):33–50.
- Moon, S. J., Jeon, J.-J., Lee, J. S. H., and Kim, Y. (2021). Learning multiple quantiles with neural networks. *Journal of Computational and Graphical Statistics*, 30(4):1238–1248.
- Vidal-Llana, X., Salort Sánchez, C., Coia, V., and Guillén, M. (2022). Noncrossing dual neural network: Joint value at risk and conditional tail expectation estimations with non-crossing conditions. *Documents de Treball* (*IREA*), 2022(15):1.
- Yu, K., Lu, Z., and Stander, J. (2003). Quantile regression: applications and current research areas. *Journal of the Royal Statistical Society: Series* D (The Statistician), 52(3):331–350.

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Thank you! Any questions?

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