Non-crossing neural network quantile regression estimation for driving data with telematics

Xenxo Vidal-Llana[1,2], Montserrat Guillén[1]

[1]Universitat de Barcelona
RiskCenter
Barcelona, Spain

Universitat de Barcelona
Facultat d'Economia i Empresa

Insurance
Data
Science

[2] juanjose.vidal@ub.edu
June 15-16, 2023

Outline

1. Motivation
2. Dataset
3. Results
4. Conclusions
5. Appendix

Risk evaluation

- **Evaluation of heavy tailed distributions is a crucial part of risk assessment.**

Risk evaluation

- **Evaluation of heavy tailed distributions is a crucial part of risk assessment.**
- When datasets present **long conditional tails** on their response variables, algorithms based on **Quantile Regression** have been widely used to assess extreme quantile behaviors.

Value at Risk (VaR) and Conditional Tail Expectation (CTE)

Definition

$$\text{VaR}_q(Y) = \inf\{y \in \mathbb{R} | F_Y(y) > q\} = F_Y^{-1}(1-q)$$

where F_Y is the distribution function of the random continuous variable Y.

Value at Risk (VaR) and Conditional Tail Expectation (CTE)

Definition

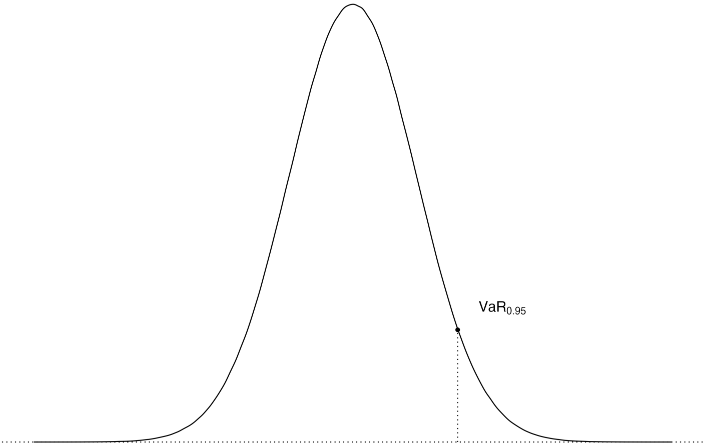
$$\text{VaR}_q(Y) = \inf\{y \in \mathbb{R} | F_Y(y) > q\} = F_Y^{-1}(1-q)$$

where F_Y is the distribution function of the random continuous variable Y.

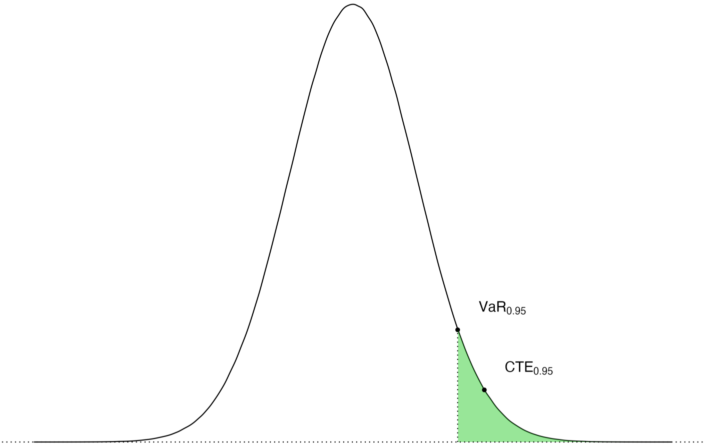
Definition

$$\text{CTE}_q(Y) = \mathbb{E}[Y | Y \geq \text{VaR}_q(Y)]$$

Value at Risk (VaR) and Conditional Tail Expectation (CTE)



Value at Risk (VaR) and Conditional Tail Expectation (CTE)



Elicitability of CTE

- The definition of elicitability can be reduced into the **existence of a scoring function** that is strictly consistent (Gneiting (2011)).

Elicitability of CTE

- The definition of elicitability can be reduced into the **existence of a scoring function** that is strictly consistent (Gneiting (2011)).
- Acerbi and Szekely (2014) found a consistent scoring function but did not opened the discussion of elicitability.

Elicitability of CTE

- The definition of elicitability can be reduced into the **existence of a scoring function** that is strictly consistent (Gneiting (2011)).
- Acerbi and Szekely (2014) found a consistent scoring function but did not opened the discussion of elicitability. Afterwards, Fissler and Ziegel (2016) prove that **CTE alone it is not elicitable, but the pair (VaR, CTE) is**.

Scoring functions

Scoring Function - VaR (Koenker and Bassett Jr (1978))

$$\rho_q(r_1, y) = (q - \mathbb{1}_{\{y - r_1 < 0\}})(y - r_1)$$

Scoring functions

Scoring Function - VaR (Koenker and Bassett Jr (1978))

$$\rho_q(r_1, y) = (q - \mathbb{1}_{\{y - r_1 < 0\}})(y - r_1)$$

Scoring Function - CTE (Fissler and Ziegel (2016))

$$S_q(r_1, r_2, y) = \mathbb{1}_{\{y > r_1\}}\big(- G_1(r_1) + G_1(y) - G_2(r_2)(r_1 - y)\big) + (1 - q)\big(G_1(r_1) - G_2(r_2)(r_2 - r_1) + \mathcal{G}_2(r_2)\big)$$

with G_1 being an increasing function, $\mathcal{G}_2$ an increasing and concave function and $\mathcal{G}_2' = G_2$

Non-crossing algorithms 1

- Acerbi and Szekely (2014) realized a non-crossing problem between the VaR and the CTE

Non-crossing algorithms 1

- Acerbi and Szekely (2014) realized a non-crossing problem between the VaR and the CTE, namely $\mathbf{VaR_q}(\mathbf{y_i}) \leq \mathbf{CTE_q}(\mathbf{y_i})$.

Non-crossing algorithms 1

- Acerbi and Szekely (2014) realized a non-crossing problem between the VaR and the CTE, namely $\mathbf{VaR_q(y_i)} \leq \mathbf{CTE_q(y_i)}$.
- **But what about several quantile levels?**

Non-crossing algorithms 2

- The problem of crossing quantiles' line of research was created by He (1997) and Yu et al. (2003)

Non-crossing algorithms 2

- The problem of crossing quantiles' line of research was created by He (1997) and Yu et al. (2003), namely $\mathbf{VaR_{q_0}(y_i)} \leq \mathbf{VaR_{q_1}(y_i)}$

Non-crossing algorithms 2

- The problem of crossing quantiles' line of research was created by He (1997) and Yu et al. (2003), namely $\mathbf{VaR_{q_0}}(\mathbf{y_i}) \leq \mathbf{VaR_{q_1}}(\mathbf{y_i})$
- Recent advances use neural networks (see Cannon (2018) and Moon et al. (2021))

Non-crossing algorithms 2

- The problem of crossing quantiles' line of research was created by He (1997) and Yu et al. (2003), namely $\mathbf{VaR_{q_0}}(\mathbf{y_i}) \leq \mathbf{VaR_{q_1}}(\mathbf{y_i})$
- Recent advances use neural networks (see Cannon (2018) and Moon et al. (2021))
- Vidal-Llana et al. (2022) presented an approach to a multiple quantile levels for VaR and CTE estimation with non-crossing conditions

Objective

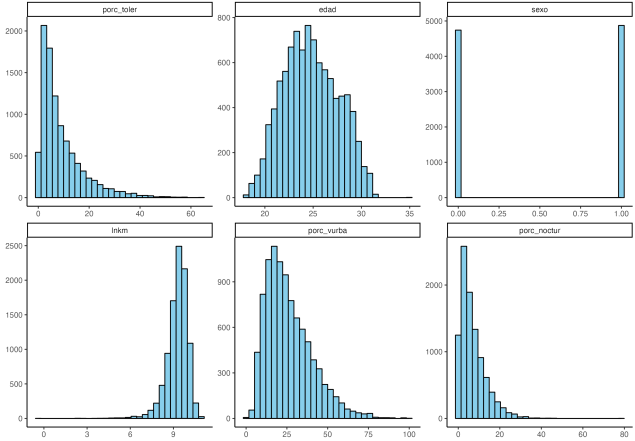
- Showcase the problematic of crossing quantiles across VaRs and between a VaR and its CTE under a telematics context

Objective

- Showcase the problematic of crossing quantiles across VaRs and between a VaR and its CTE under a telematics context
- Compare a classical approach to several quantile levels against a methodology that assures non-crossing conditions

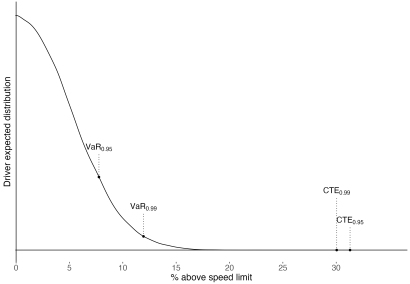
Dataset

Telematic information from year 2015 of 9,614 drivers from a Spanish insurance company

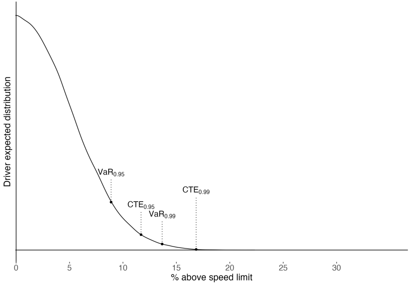


Crossing example

Two Step



NCDNN



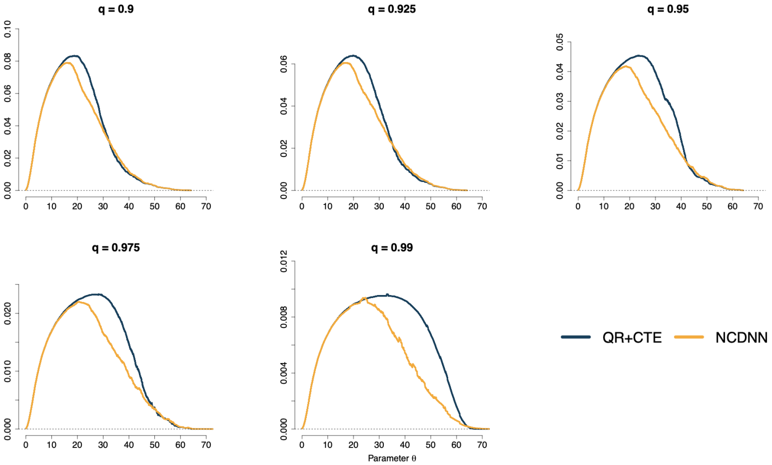
Crossings on a Two Step approach

q_i - q_{i+1}	0.9 - 0.925	0.925 - 0.95	0.95 - 0-975	0.975 - 0.99
$VaR_{q_i} > VaR_{q_{i+1}}$	3 (0%)	1 (0%)	2 (0%)	2 (0%)

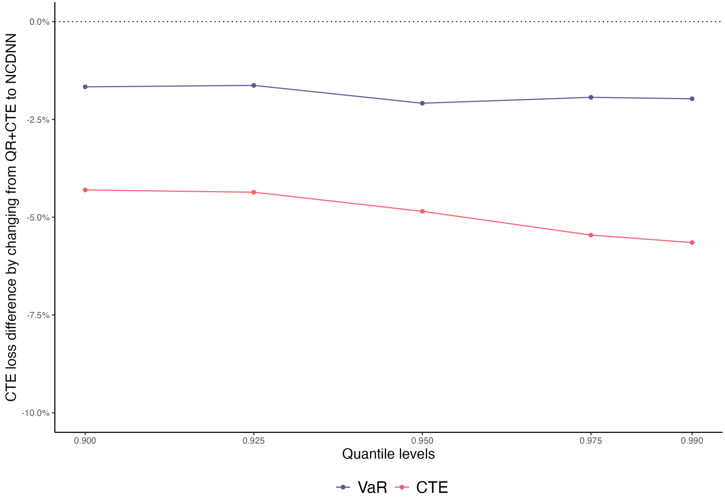
Crossings on a Two Step approach

q_i - q_{i+1}	0.9 - 0.925	0.925 - 0.95	0.95 - 0-975	0.975 - 0.99
$VaR_{q_i} > VaR_{q_{i+1}}$	3 (0%)	1 (0%)	2 (0%)	2 (0%)
$CTE_{q_i} > CTE_{q_{i+1}}$	0 (0%)	541 (6%)	1,560 (16%)	176 (2%)

Murphy Diagrams: CTE comparison



Loss improvement



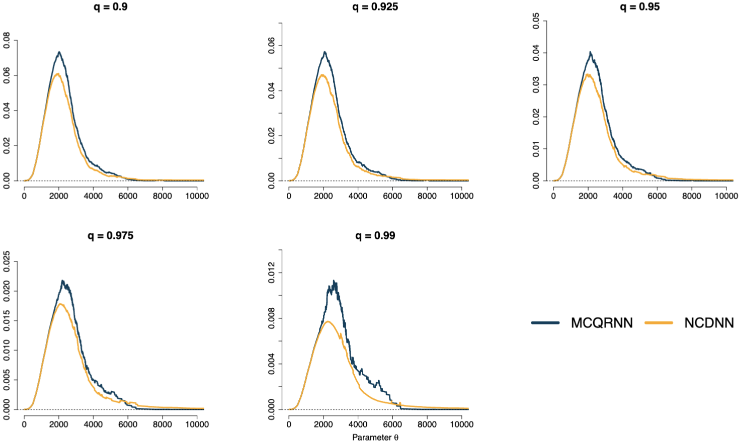
Conclusions

- Inside an insurance company pricing scheme, **crossing predictions become unfeasible estimations**, thus the usefulness of non-crossing algorithms

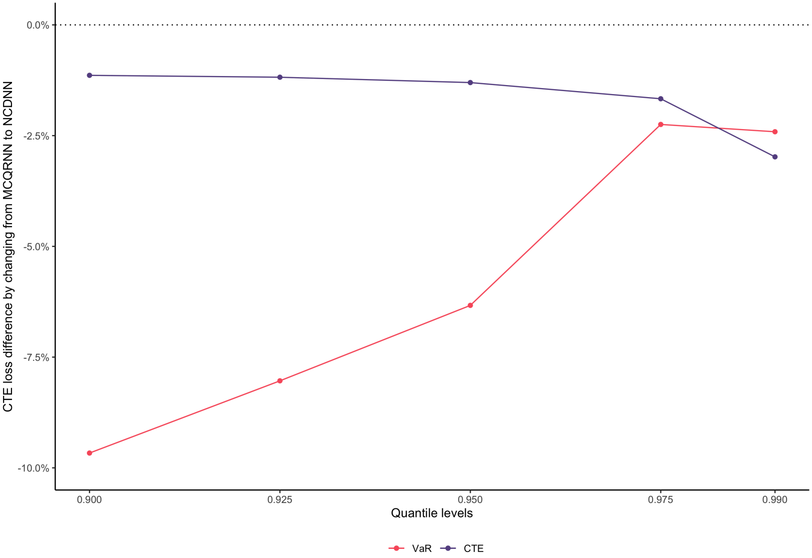
Conclusions

- Inside an insurance company pricing scheme, **crossing predictions become unfeasible estimations**, thus the usefulness of non-crossing algorithms
- For financial practitioners, and after Basel III recommendations, **non-crossing predictions help assess bank reserves in a more consistent way**

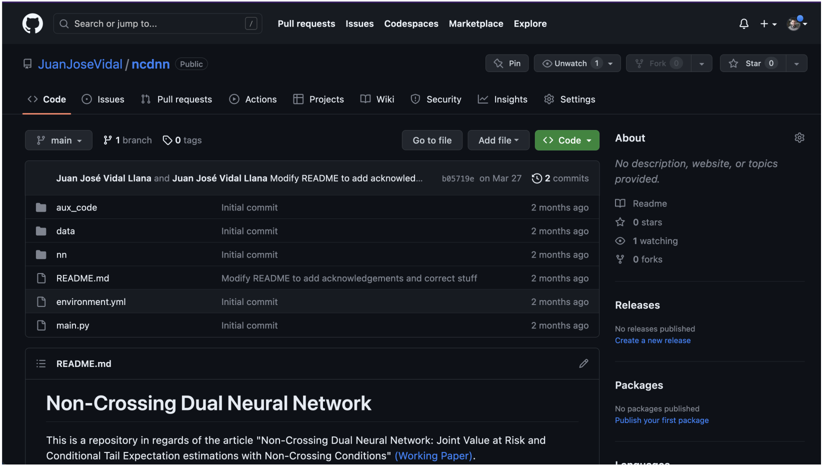
Additional results I



Additional results II



GitHub Repository



GitHub NCDNN Repository

Special acknowledgements

- Spanish Ministry of Science: "PID2019-105986GB-C21/AEI/10.13039/501100011033"
- NextGenerationEU: "TED2021-130187B-I00"

References

Acerbi, C. and Szekely, B. (2014). Back-testing expected shortfall. *Risk*, 27(11):76–81.

Cannon, A. J. (2018). Non-crossing nonlinear regression quantiles by monotone composite quantile regression neural network, with application to rainfall extremes. *Stochastic Environmental Research and Risk Assessment*, 32(11):3207–3225.

Fissler, T. and Ziegel, J. F. (2016). Higher order elicitability and Osband's principle. *The Annals of Statistics*, 44(4):1680–1707.

Gneiting, T. (2011). Making and evaluating point forecasts. *Journal of the American Statistical Association*, 106(494):746–762.

He, X. (1997). Quantile curves without crossing. *The American Statistician*, 51(2):186–192.

List of References

Koenker, R. and Bassett Jr, G. (1978). Regression quantiles. *Econometrica: Journal of the Econometric Society*, 46(1):33–50.

Moon, S. J., Jeon, J.-J., Lee, J. S. H., and Kim, Y. (2021). Learning multiple quantiles with neural networks. *Journal of Computational and Graphical Statistics*, 30(4):1238–1248.

Vidal-Llana, X., Salort Sánchez, C., Coia, V., and Guillén, M. (2022). Non-crossing dual neural network: Joint value at risk and conditional tail expectation estimations with non-crossing conditions. *Documents de Treball (IREA)*, 2022(15):1.

Yu, K., Lu, Z., and Stander, J. (2003). Quantile regression: applications and current research areas. *Journal of the Royal Statistical Society: Series D (The Statistician)*, 52(3):331–350.

Thank you! Any questions?