Causal Inference and Fairness in Insurance Pricing

Insurance Data Science Conference 2023

Research by

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What is fairness?

Unfair discrimination for ratemaking

“**A rate is** reasonable and **not** excessive, inadequate, or **unfairly discriminatory if it is an actuarially sound estimate of the expected value of all future costs** associated with an individual risk transfer.”

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The debate regarding the formal definition of fairness never really settled (Frezal and Barry, 2020) , and the rise of machine learning and big data increases the potential for harm due to unfairness (Embrechts and Wüthrich, 2022).

Why should we care about fairness?

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3. Maintaining public trust

Causal inference explained

Why causal inference and fairness?

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Fairness in insurance:
Estimate a **premium** while avoiding **unfair** bias from **prohibited** confounders.

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- Causal inference offers a deeper understanding of relationships in a dataset **(True risk factors, Araiza Iturria et al., 2022)**.
- Causal inference allows to answer questions that goes beyond the observable dataset **("What would have been the premium, had there been no disparity?")**.

Reviewing the discrimination-free formula

Important definitions

Table 1: Key Definitions

Notation	Description	Example
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$$
\begin{tikzcd}
D \arrow[d] \arrow[dr] & \\
X \arrow[r] & Y
\end{tikzcd}
$$

Figure 1: Typical directed acyclic graph (DAG) of fairness in insurance

Discrimination-free formula

Recall the discrimination-free premium proposed by Lindholm et al. (2022) with the real world measure $\mathbb{P}$:

$$\mu^{DF}(X) = \int_d \mathbb{E}\left[Y|X, D\right] \mathrm{d}\mathbb{P}\left(D = d\right).$$

Counterfactual identification

Counterfactual for a discrimination-free premium

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$$\mathbb{E}\left[Y^x\right] \overset{\text{Causal assumpt.}}{=} \underbrace{\mathbb{E}_D\left\{\mathbb{E}\left[Y|X,D\right]\right\} = \mu^{DF}(X)}_{\text{Discrimination-free formula}}$$

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1. The prohibited attribute has to be a **confounder**.

$$\begin{array}{ccc} D & & \\ \downarrow & \searrow & \\ X & \longrightarrow & Y \end{array}$$

Figure 2: Desired DAG to satisfy assumptions

Counterfactual for a discrimination-free premium : remark

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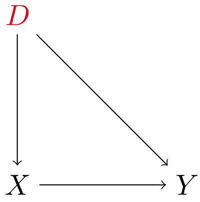
2. Positivity, exchangeability and consistency must be valid.



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Inverse probability weighting

Towards a propensity score weighting

We start again with the discrimination-free formula, focusing on the weighting term:

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Multiplying by a real fraction equal to 1, we obtain :

$$\mu^{IPW}(X) = \int_d \mathbb{E}\left[Y|X, D=d\right] \frac{\mathrm{d}\mathbb{P}\left(D=d|X\right)}{\mathrm{d}\mathbb{P}\left(D=d|X\right)} \mathrm{d}\mathbb{P}(D=d)$$

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&= \int_d \mathbb{E}\left[Y \frac{\mathrm{d}\mathbb{P}(D = d)}{\mathrm{d}\mathbb{P}\left(D = d|X\right)} \middle| X, D = d\right] \mathrm{d}\mathbb{P}\left(D = d|X\right).
\end{aligned}$$

Weights for fairness

We get a weight w that introduces fairness in our discrimination-free formula :

$$\mu^{IPW}(X) = \mathbb{E}_D \left\{ \mathbb{E}\left[Y \frac{\mathrm{d}\mathbb{P}(D)}{\mathrm{d}\mathbb{P}\left(D|X\right)} \middle| X, D \right] \middle| X \right\}$$

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A variety of weights (see, e.g. Li and Li, 2019) and a variety of estimators (See Fong et al., 2018, for non-parametric estimator) exist.

Intuitive properties of weights

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They do not distort variables on the aggregate level :

$$\mathbb{E}\left[w\right] = 1$$
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$$\mathbb{E}\left[X \cdot w\right] = \mathbb{E}\left[X\right]$$
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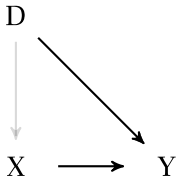

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For every profile i :

- x_i : Occupation (Nursing ou Mechanic)
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Table 2: Dataset of motor vehicle claims

i	x_i	d_i	e_i	Y_i
1	💉	F	95	50
2	💉	M	5	200
3	🔧	F	10	75
4	🔧	M	90	250

Weight calculation

We use formula mentioned previously for weights :

$$w_1 = \frac{\hat{\mathrm{P}}(D = \mathtt{F})}{\hat{\mathrm{P}}(D = \mathtt{F} | X = \text{syringe})} = \frac{\left(\frac{95 + 10}{200}\right)}{\left(\frac{95}{95 + 5}\right)} \approx 0.5526$$

Table 3: Required information for weight calculation

i	x_i	d_i	e_i	★ w_i ★
1	syringe	F	95	
2	syringe	M	5	
3	wrench	F	10	
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$$w_3 = \frac{\hat{\mathrm{P}}(D = \mathtt{F})}{\hat{\mathrm{P}}(D = \mathtt{F} | X = \text{🔧})} = \frac{\left(\frac{95+10}{200}\right)}{\left(\frac{10}{10+90}\right)} = 5.25$$

$$w_4 = \frac{\hat{\mathrm{P}}(D = \mathtt{M})}{\hat{\mathrm{P}}(D = \mathtt{M} | X = \text{🔧})} = \frac{\left(\frac{90+5}{200}\right)}{\left(\frac{90}{10+90}\right)} \approx 0.5278$$

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1	💉	F	95	0.5526
2	💉	M	5	9.5
3	🔧	F	10	5.25
4	🔧	M	90	0.5278

New exposure

Table 4: Dataset of motor vehicle claims

i	x_i	d_i	e_i	w_i	e_i^*	Y_i
1		F	95	0.5526	52.5	50
2		M	5	9.5	47.5	200
3		F	10	5.25	52.5	75
4		M	90	0.5278	47.5	250

With

$$e_i \cdot w_i = e_i^*$$

Avoiding D using different approaches

Table 5: Aggregated profiles using e

x_j	e_j	$\mu^{U}(X)$ (Unaware)	$\mu^{IPW}(X)$ (Inverse pribability weighting)
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	$95 + 5 =$ 100	$0.95 \cdot 50 +$ $0.05 \cdot 200 =$ 57.5	
	$10 + 90 =$ 100	$0.10 \cdot 75 +$ $0.90 \cdot 250 =$ 232.5	

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	$95+5=$ 100	$0.95 \cdot 50 +$ $0.05 \cdot 200 =$ 57.5	$0.525 \cdot 50 +$ $0.475 \cdot 200 =$ 121.25
	$10+90=$ 100	$0.10 \cdot 75+$ $0.90 \cdot 250 =$ 232.5	$0.525 \cdot 75+$ $0.475 \cdot 250 =$ 158.125

Conclusion

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- Standardization (g-formula) **(Pope and Sydnor, 2011; Aseervatham et al., 2016; Lindholm et al., 2022; Araiza Iturria et al., 2022)**
- Inverse probability weighting (IPW) **(Lindholm et al., 2023)**
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- Inverse probability weighting (IPW) **(Lindholm et al., 2023)**
- Matching (optimal transport) **(Charpentier et al., 2023; Lindholm et al., 2023).**

There is still some work to apply causal inference with high-dimensional X and D (Li and Li, 2019).

Thank you

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