A neural network approach for selecting efficient reinsurance strategies

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Agenda

1. Introduction
2. Problem and perimeter of the analysis
3. Neural network approach for selecting efficient reinsurance strategies
4. Conclusions

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Introduction

Motivation and background

- The choice of optimal strategies in an uncertain setting is a topic object of extensive academic and professional studies
- In the insurance context one relevant area of application regards the analysis of optimal reinsurance strategies
- One relevant complexity in determining the efficient frontier in related with the fact that:
  - The more we increase the elements defining the reinsurance strategies the more we increase the number of combinations to analyze
  - The higher the precision we want to achieve the higher the number of combinations to analyze
- We define an approach for obtaining efficient reinsurance strategies in a multi-objective optimization framework for a non-life insurance company, which allows to analyze complex reinsurance strategies with high precision and requiring limited data

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Problem and perimeter of the analysis

Risk reserve equation

- We extend the risk reserve equation to the most general case, considering multiple lines of business, multiple reinsurers with different ratings and reinsurance treaties

Multiple lines of business

Multiple reinsurers

$$\tilde{U}_{t+1} = U_t(1+j) + \sum_{l=1}^{L}\left[\left(B_{l,t+1} - \tilde{X}_{l,t+1} - E_{l,t+1}\right) - \left(B^{re}_{l,t+1} - \sum_{r=1}^{R}\left(\tilde{X}^{re,d\,(r)}_{l,t+1} + \tilde{C}^{re\,(r)}_{l,t+1}\right)\right)\right](1+j)^{0.5}$$

Gross technical result

Effect of counterparty default

Effect of reinsurance treaties

Problem and perimeter of the analysis

Efficient frontier

- From the risk reserve equation we define two metrics of interest for the insurance company
- As it is usual the case in practice, the insurer wants to simultaneously maximize its profitability and minimize its risk. In this context we chose the following metrics:

  Expected Return on Equity (RoE): $\mathrm{E}\left(\widetilde{RoE}\right) = \frac{E(\tilde{U}_{t+1})}{U_t} - 1$

  Solvency Ratio (SR): $\mathrm{SR} = \frac{U_t}{-q_{0,5\%}(\tilde{U}_{t+1} - U_t)}$

- For calculating the Solvency Ratio we assume that the own funds are equal to the initial capital U_t, while the capital required is equal to the quantile of the risk reserve at the end of time *t+1* at 99.5% confidence level

Problem and perimeter of the analysis

Efficient frontier: optimization problem

- The objective is to find the reinsurance strategies that jointly maximize both the objective metrics. Hence, we define a preference structure for choosing between different strategies:

Preference structure:

- We say that strategy S_1 dominates strategy S_2 if it is at least better in one objective function and equal in all the others:

$$S_1 \succ S_2$$
$$if\ E(\widetilde{RoE})(S_1) > E(\widetilde{RoE})(S_2)\ \wedge SR(S_1) \geq SR(S_2)\ or$$
$$if\ E(\widetilde{RoE})(S_1) \geq E(\widetilde{RoE})(S_2)\ \wedge SR(S_1) > SR(S_2)$$

- The strategies that are not dominated by any other are called efficient and they define the Pareto Frontier

Optimization problem:

- Formally we have the following optimization problem:

$$max_S\left(E\left(\widetilde{RoE}(S)\right), SR(S)\right)\ s.t.$$
$$E\left(\widetilde{RoE}(S)\right) \geq RoE_{target}$$
$$SR(S) \geq SR_{target}$$
$$\alpha_i \leq \alpha_{i,target}$$
$$etc.$$

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Neural network approach for selecting efficient reinsurance strategies

Steps for obtaining the efficient reinsurance strategies

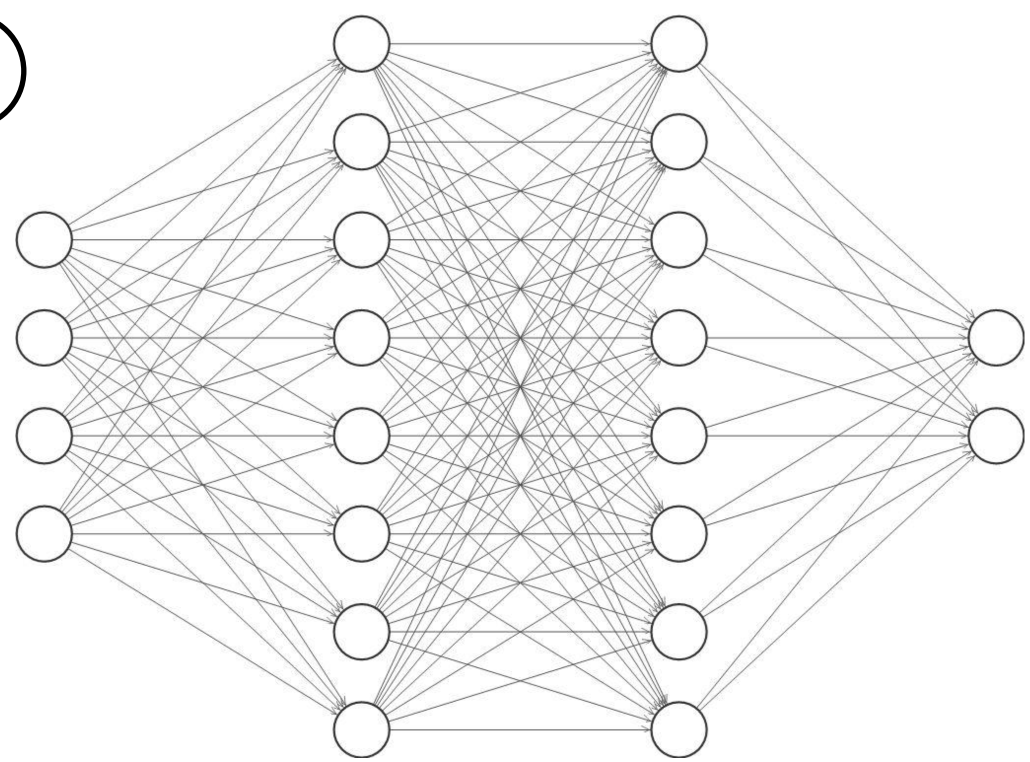
1. **Data simulation:** Application of a stochastic simulation approach for obtaining combinations of reinsurance strategies and their corresponding RoE and SR
2. **Derivation of the efficient frontier with neural network:**
   a) **Neural network model 1:** Function approximation of the relation between input (characteristics of the reinsurance treaties and of the reinsurers) and output (corresponding metrics of risk and return)
   b) **Neural network model 2:** Model which takes as input one objective metric (RoE) and returns the same metrics (RoE) while maximizing the other one (SR)
   c) **Efficient strategies determination:** Derivation of the parameters of reinsurance strategies on the efficient frontier defined by previous neural network

Neural network approach for selecting efficient reinsurance strategies

Efficient frontier with Neural Network:

1. Neural Network model for predicting output
2. Neural Network model for estimating the efficient frontier

1.



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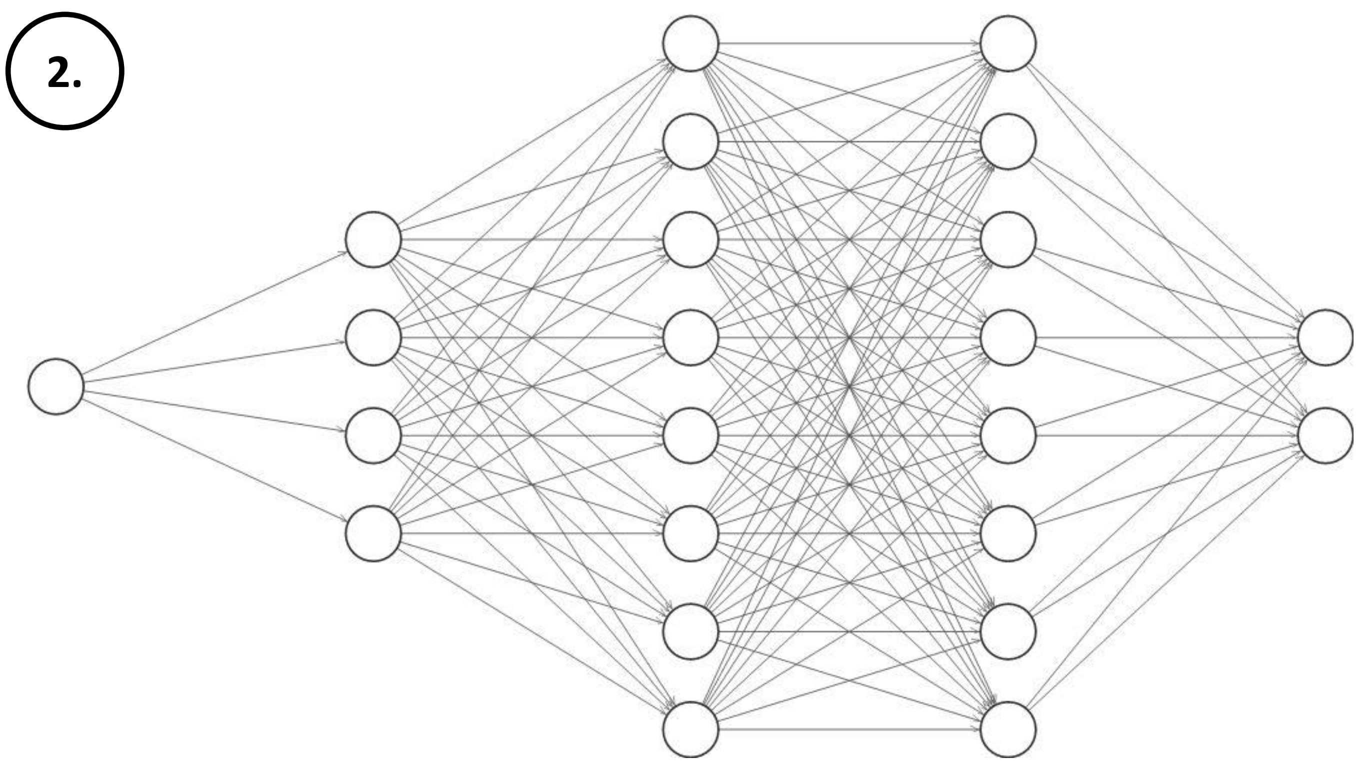

Neural network approach for selecting efficient reinsurance strategies

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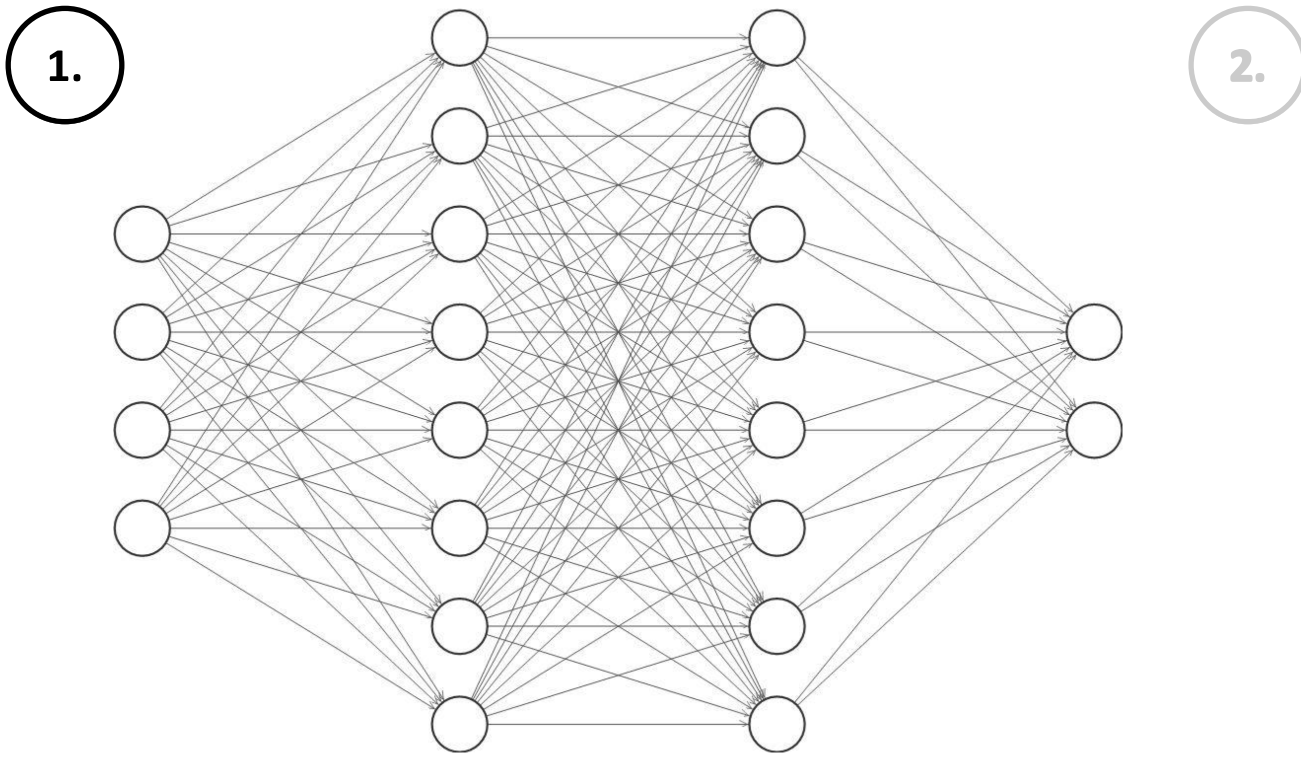
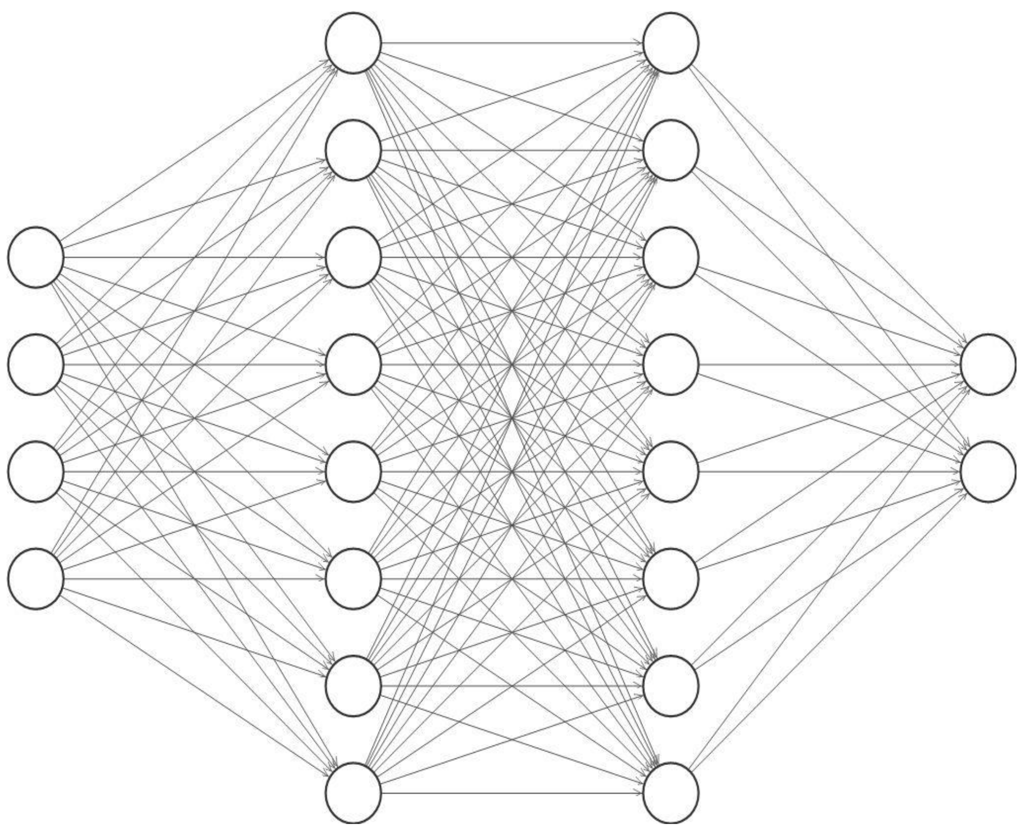


Neural network approach for selecting efficient reinsurance strategies

Efficient frontier with Neural Network:

1. Neural Network model for predicting output:
   a) Simulate a limited number of combinations of the input and obtain the corresponding metrics of Risk and Return
   b) Build a Neural Network for predicting Risk and Return, given the set of input
   c) Model and assess the performance of the Neural Network according to a chosen methodology (e. g. divide the observation in training, validation and test set, minimize the loss of training/validation set and assess the performance on the test set – which was kept separate in the training phase, etc.)



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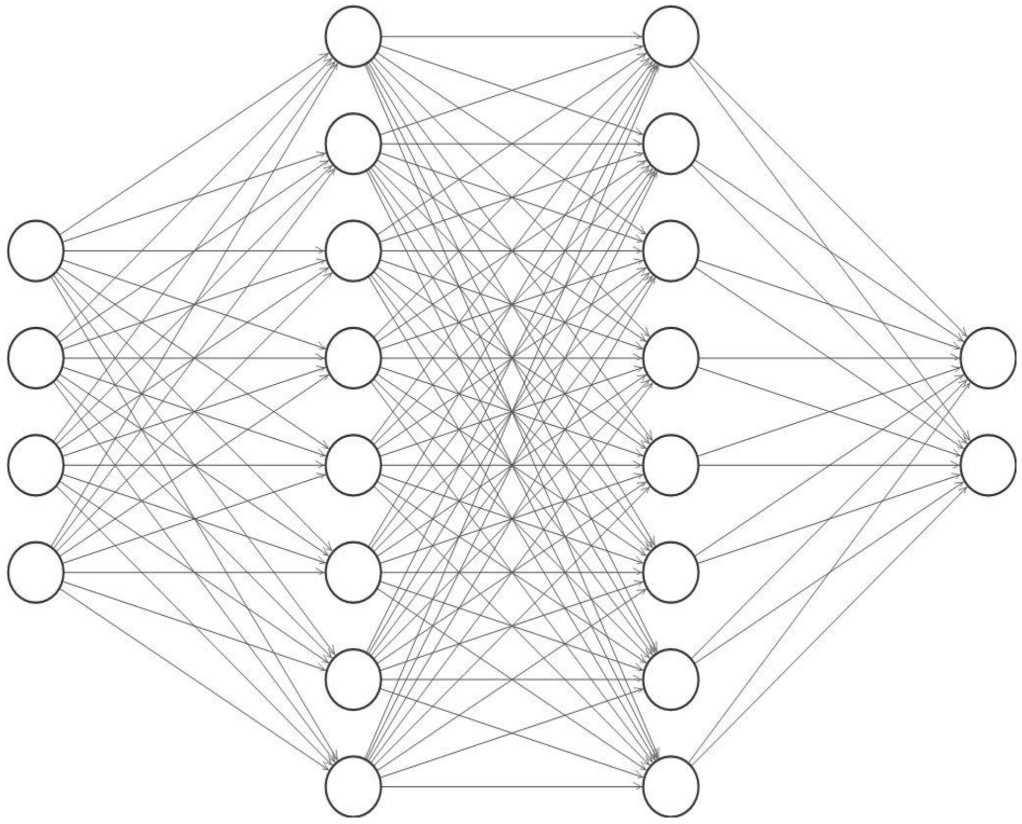

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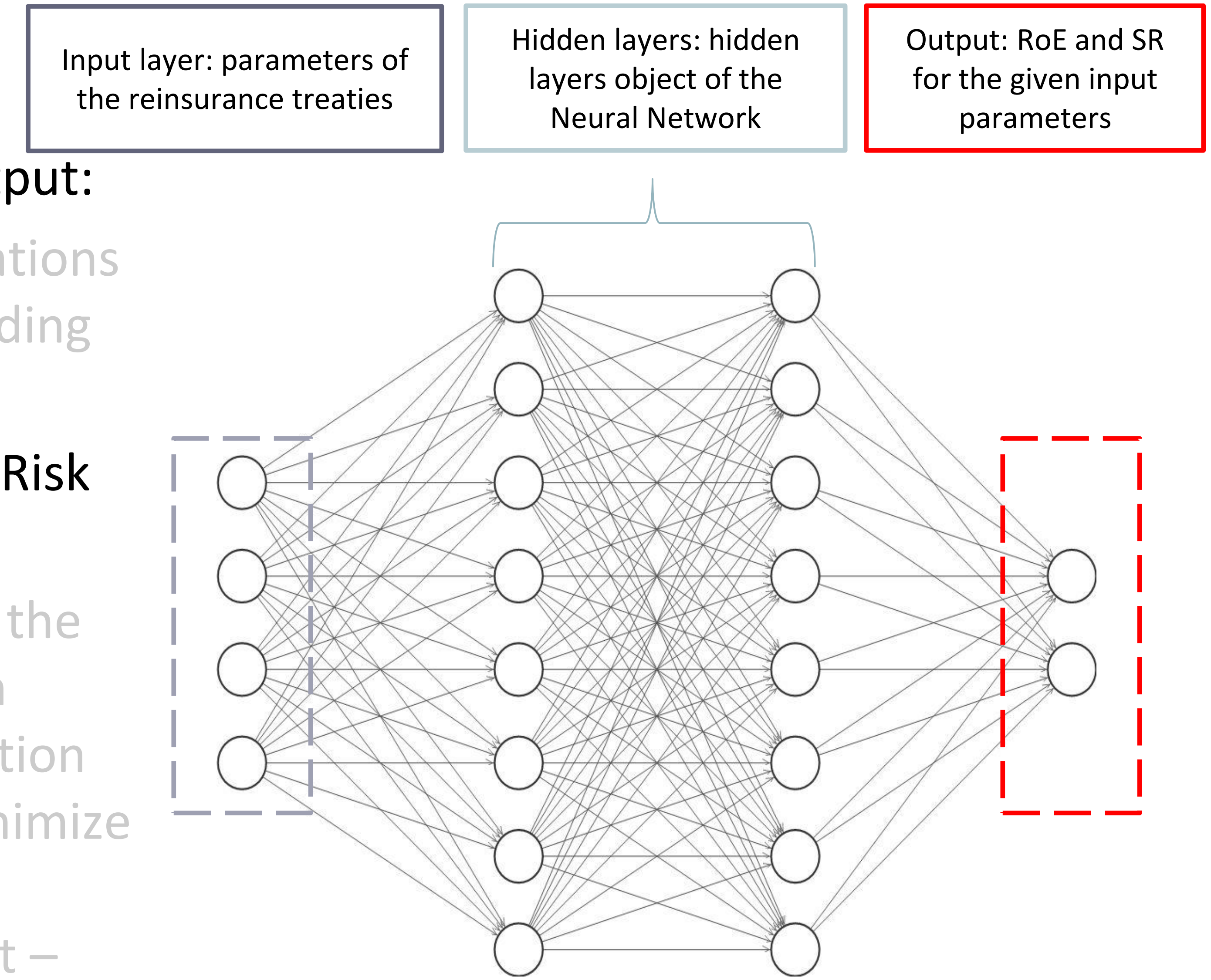

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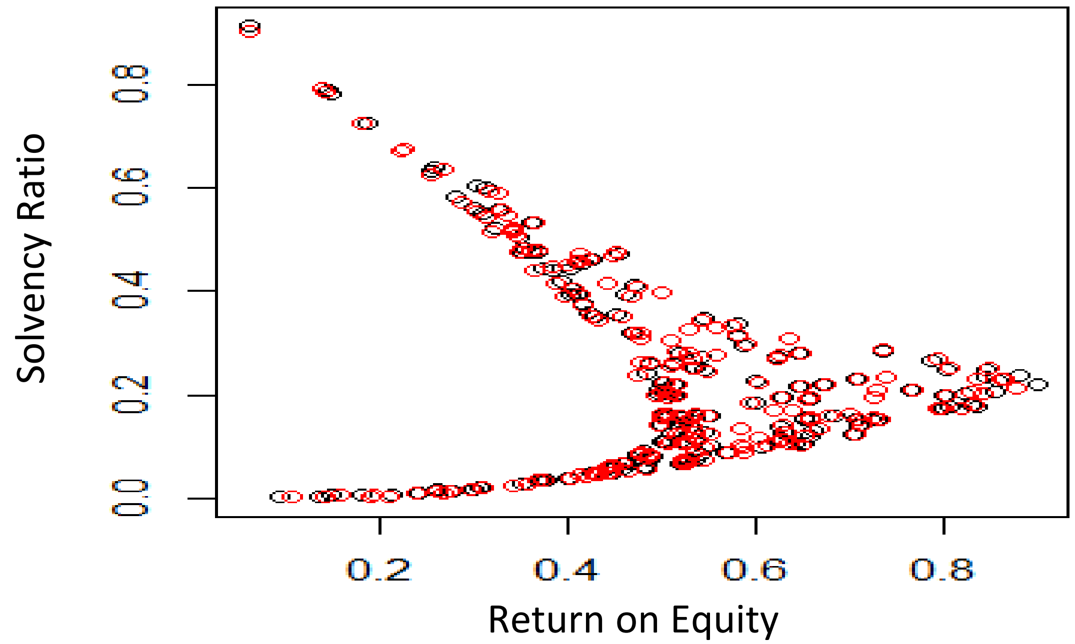
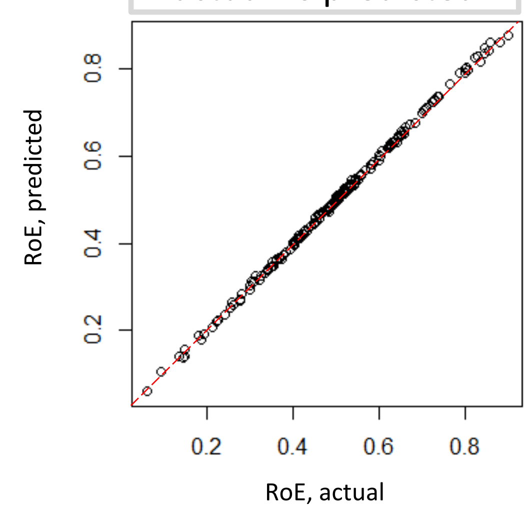
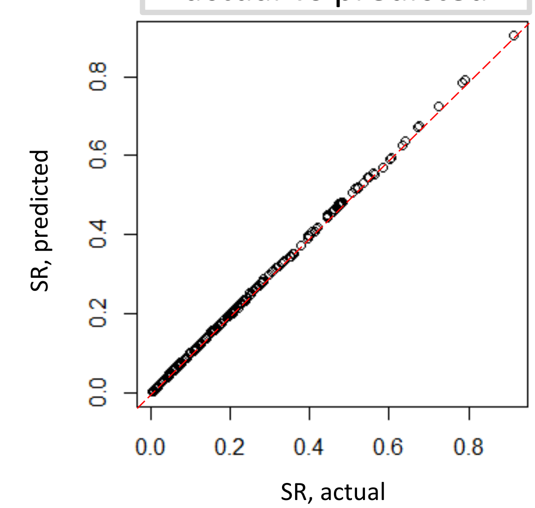
Total loss
(weighted average of RoE and SR losses)



RoE loss
actual vs predicted



SR loss
actual vs predicted



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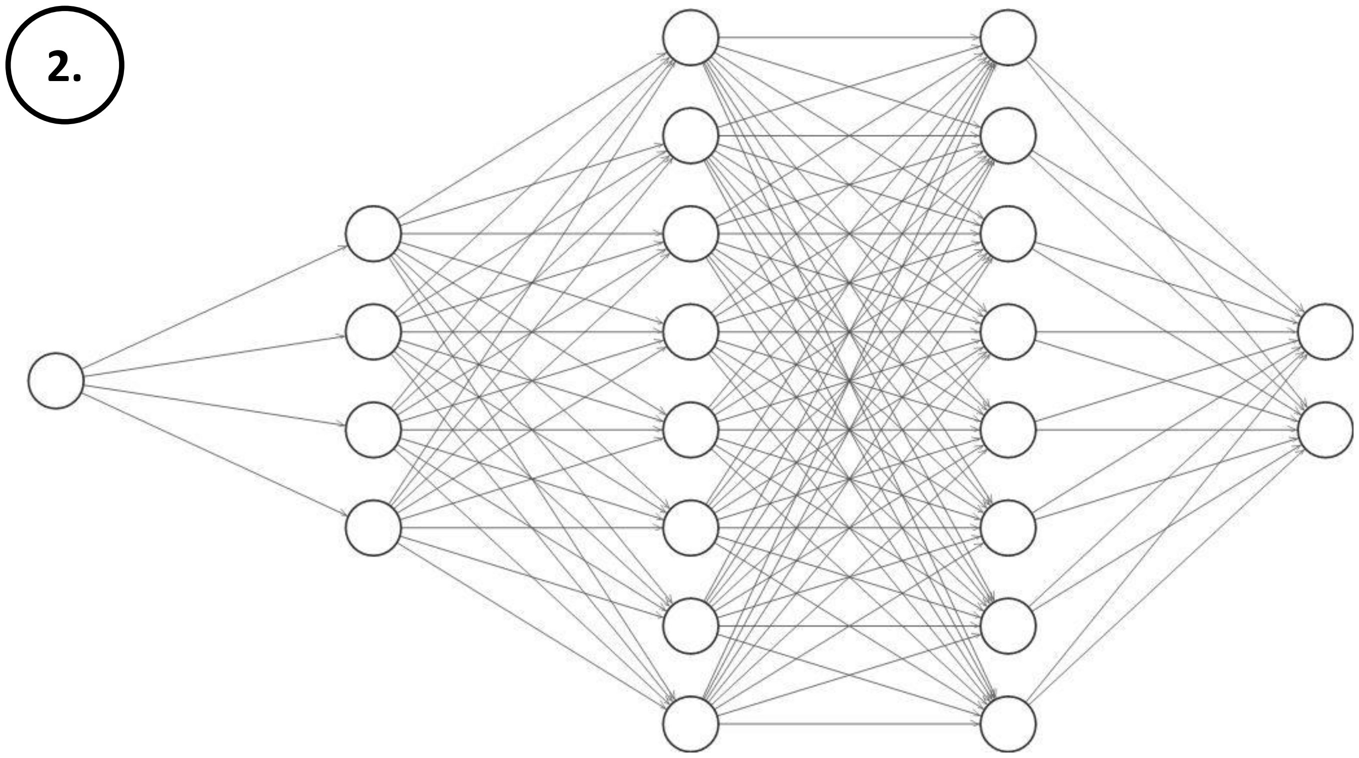
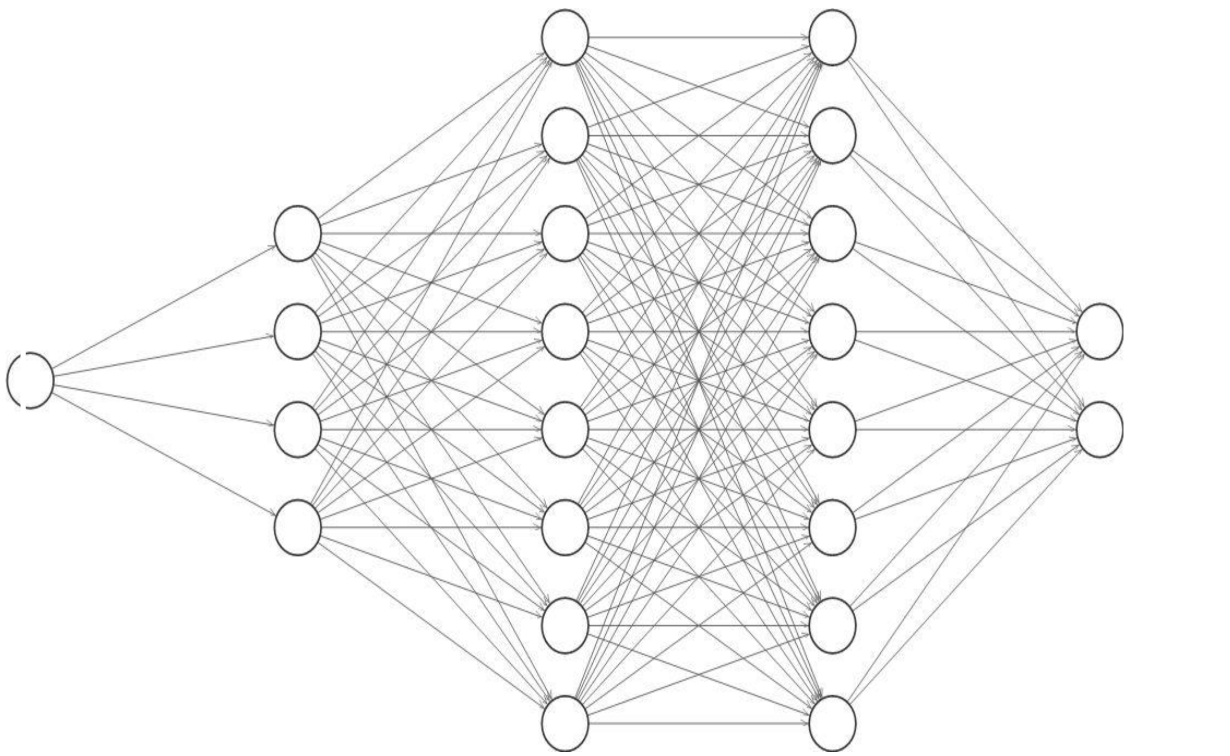


Neural network approach for selecting efficient reinsurance strategies

Efficient frontier with Neural Network:

2. Neural Network model for efficient frontier:
   a) Build a Neural Network composed as a concatenation of:
      1. A single input
      2. Neural Network built before, with all the parameters set as *not trainable*
   b) Create a list of “fake” input-output with:
      Input: random number in the domain of one of the two output (in this case RoE)
      Output: vector of random numbers:
      Output1 (RoE) := Input (RoE)
      Output2 (SR) := fixed number “big enough”
   c) Define an appropriate loss function and train the neural network
   d) Apply the Neural Network to a list of input in Output1 (RoE) domain and derive the efficient frontier



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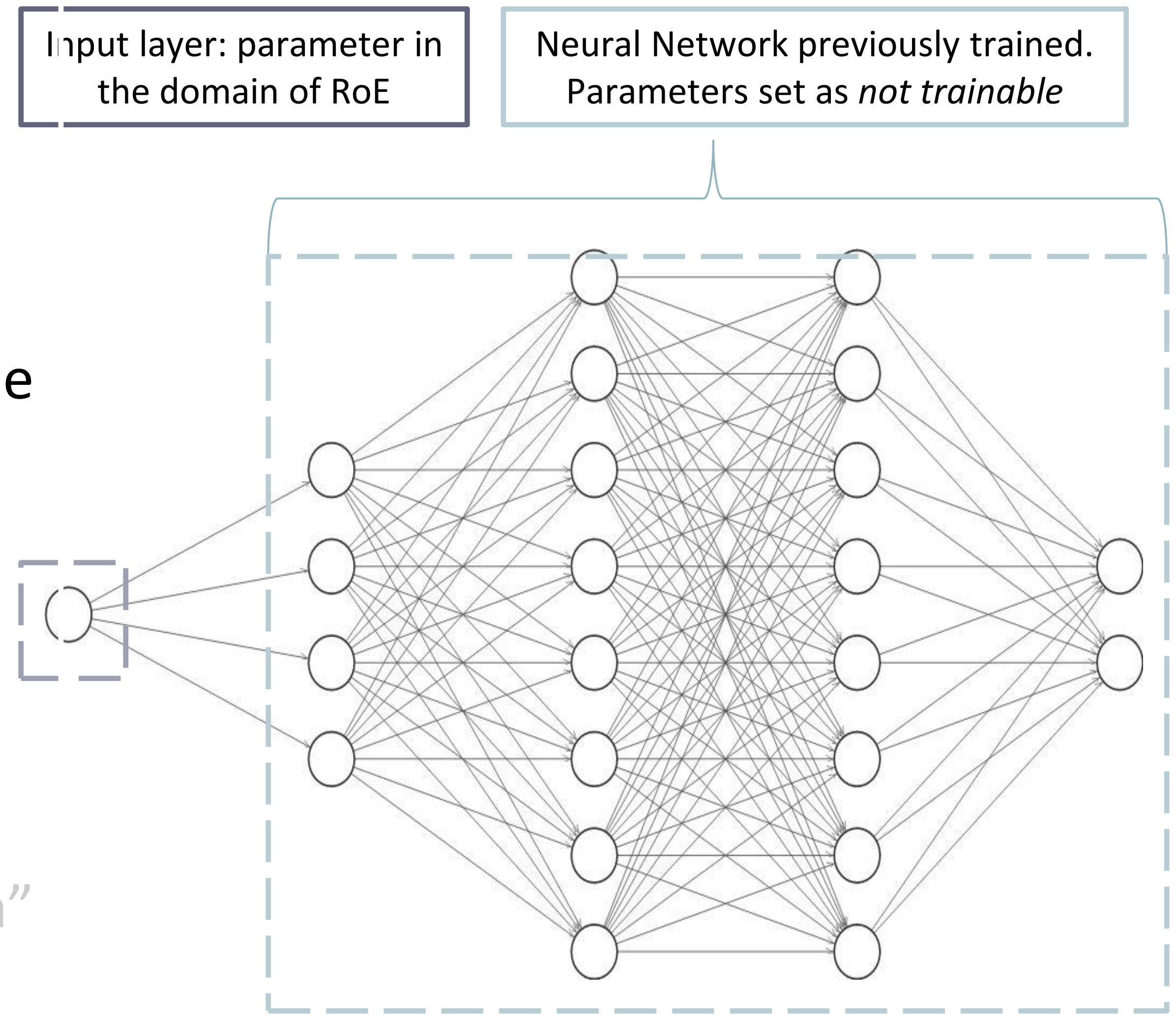

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Input: RoE	Output1: RoE	Output2: SR
-20.0%	-20.0%	500%
-18.0%	-18.0%	500%
-16.0%	-16.0%	500%
-14.0%	-14.0%	500%
…	…	…
0.0%	0.0%	500%
5.0%	5.0%	500%
6.0%	6.0%	500%
7.0%	7.0%	500%
10.0%	10.0%	500%
…	…	…
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Loss function

$$
\begin{aligned}
L &= \mathcal{L}(\mathbf{y}, \hat{\mathbf{y}}) \\
&= \mathcal{L}\left(\mathbf{y}, f\left(\mathbf{x}, \widehat{W}\right)\right) \\
&= p_1 \mathcal{L}_1(y_1, \hat{y}_1) + (1-p_1)\mathcal{L}_2(y_2, \hat{y}_2) \\
&= \underbrace{p_1 \frac{1}{n} \sum_{i=1}^{n} \max(0, y_{1,i} - \hat{y}_{1,i})}_{\text{Aim at ensuring that the predicted RoE is as close as possible to the input RoE}} - (1-p_1)\hat{y}_2
\end{aligned}
$$

Aim at ensuring that the predicted SR is as high as possible

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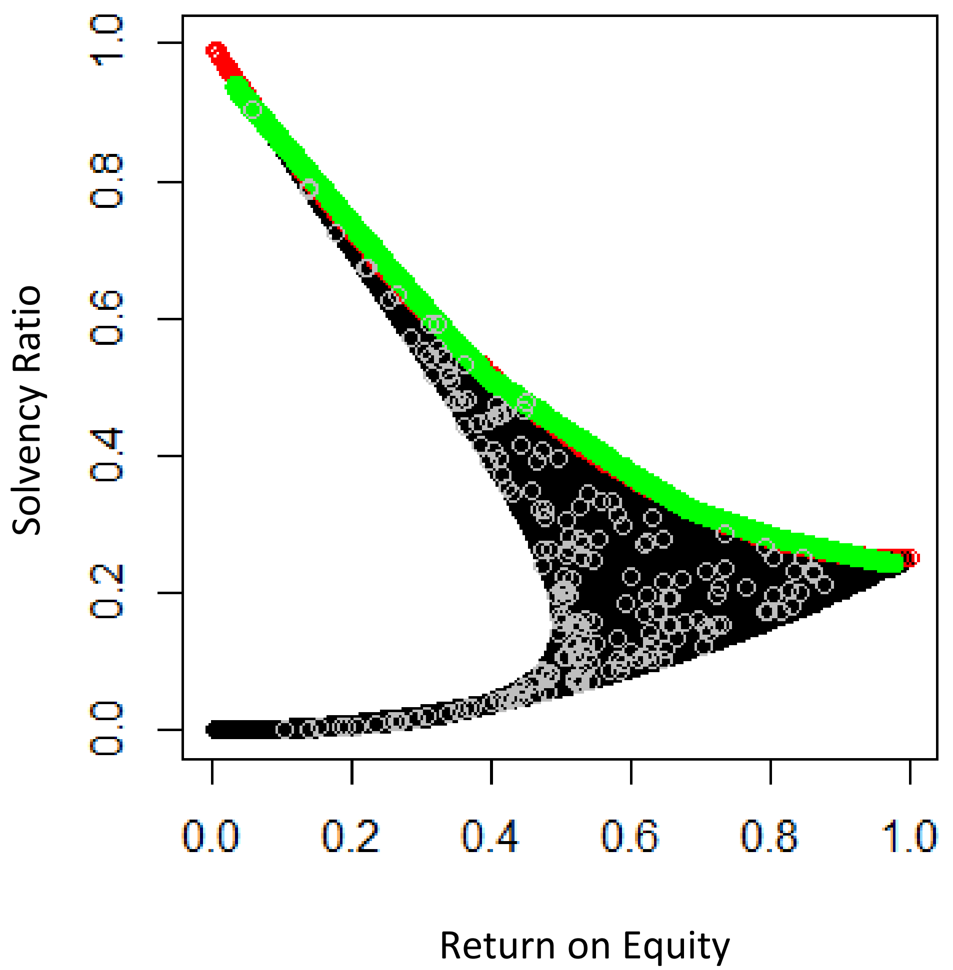


Neural network approach for selecting efficient reinsurance strategies

Numerical analysis

- We propose a numerical analysis for determining the efficient frontier in a multi-objective framework, where the insurer aims at finding the optimal reinsurance strategies using as metrics the RoE and SR

Data simulation

- We simulate 50,000 combinations of reinsurance strategies to calibrate the neural network model, with split of the dataset in training-test set (90%-10%):

  Numerical variables: sample 50,000 data from a uniform distribution on the specific support

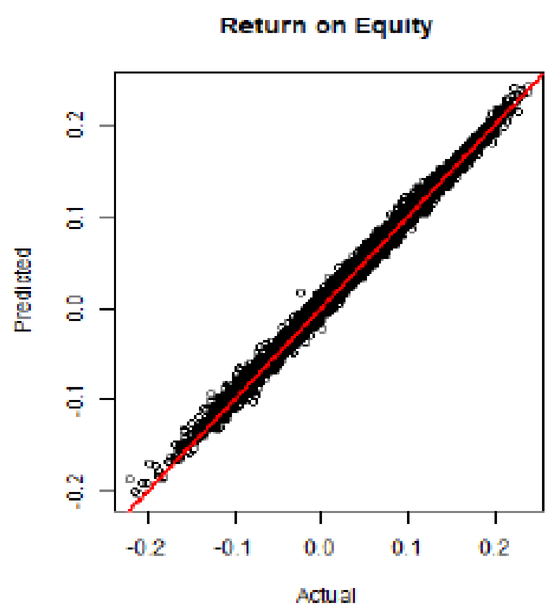
  Categorical variables: sample 50,000 data from a so-called generalized Bernoulli distribution with equal probability for each element

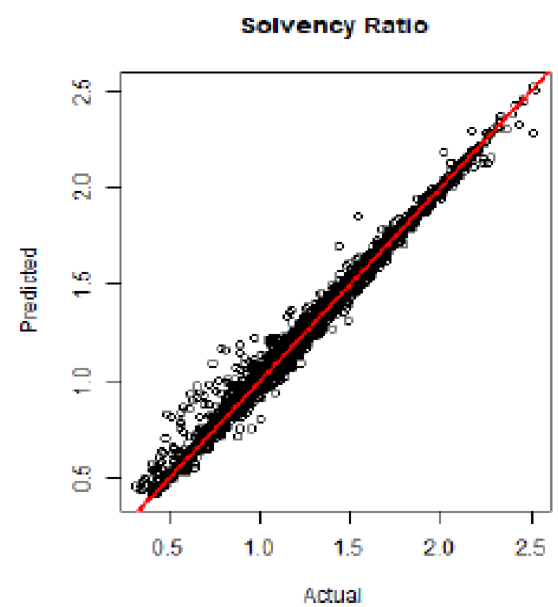
Variable	Type	Support	Description
D_{MTPL}	Numerical	$[100,000; 1,000,000]$	Deductible value of XL treaty for the MTPL segment
L_{MTPL}	Numerical	$[1,000,000; 10,000,000]$	Limit value of the XL treaty for the MTPL segment
D_{GTPL}	Numerical	$[100,000; 1,000,000]$	Deductible value of XL treaty for the GTPL segment
L_{GTPL}	Numerical	$[1,000,000; 10,000,000]$	Limit value of the XL treaty for the GTPL segment
α_{MTPL}	Numerical	$[0,1]$	Retention quota for the MTPL segment
α_{GTPL}	Numerical	$[0,1]$	Retention quota for the GTPL segment
α_{MOD}	Numerical	$[0,1]$	Retention quota for the MOD segment
$multi_{RE}$	Categorical	$\{0,1\}$	Indicator for single/ multiple reinsurer
CQS_{MTPL}	Categorical	$\{0,1,2,3,4,5,6\}$	CQS of the reinsurer(s) for the MTPL segment
CQS_{GTPL}	Categorical	$\{0,1,2,3,4,5,6\}$	CQS of the reinsurer(s) for the GTPL segment
CQS_{MOD}	Categorical	$\{0,1,2,3,4,5,6\}$	CQS of the reinsurer(s) for the MOD segment

- Input variables which defines the different reinsurance strategies
- Numerical and categorical variable, each one defined on a specific support
- First 7 variables describe the characteristics of the excess of loss and quota share treaties for each line of business
- Remaining 4 variables define the presence of single/multiple reinsurer(s) and the credit quality step of the reinsurer(s) for each LoB

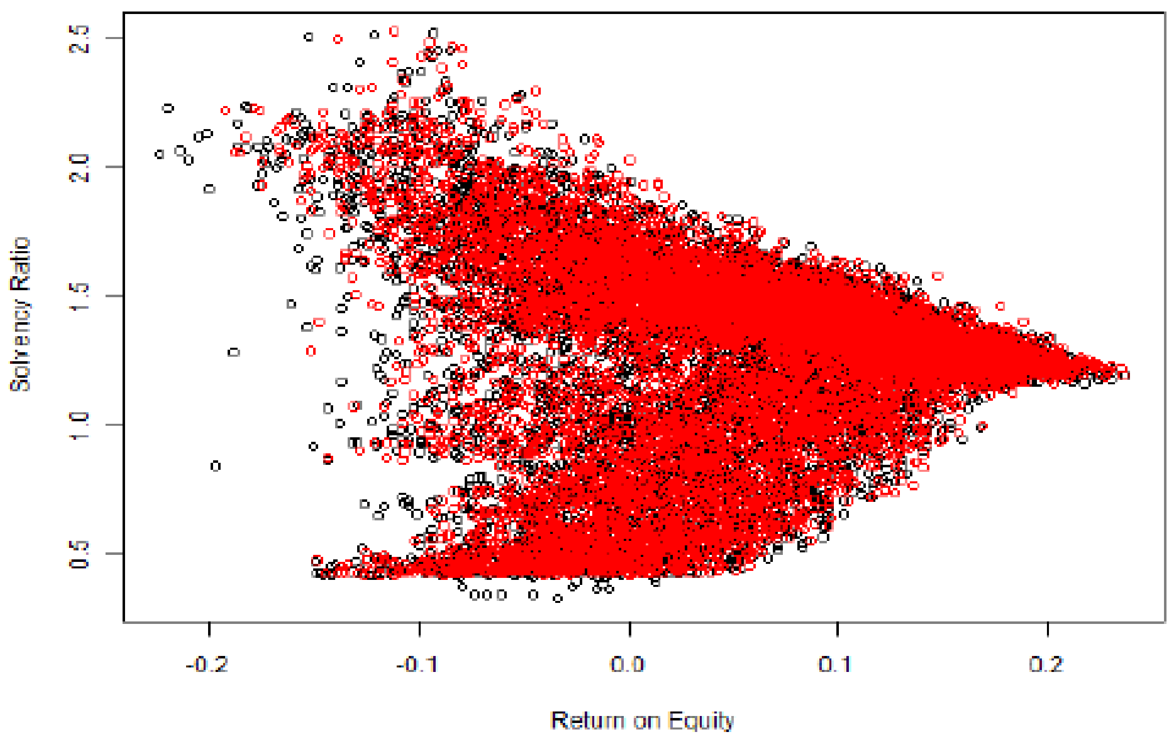
Neural network approach for selecting efficient reinsurance strategies

Neural network model 1





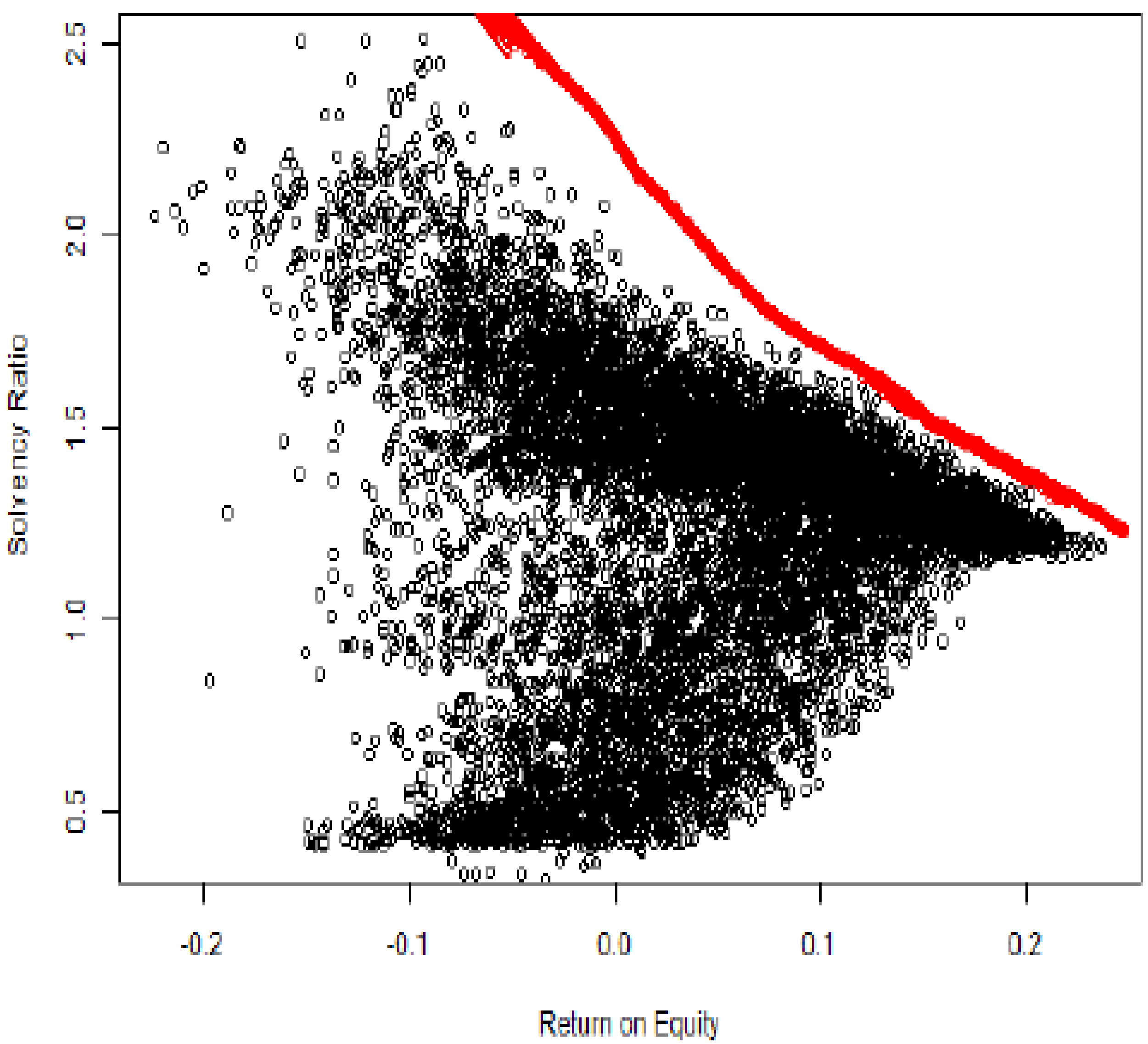
- The model shows more difficulty in estimating the actual Solvency Ratio than the expected Return on Equity
- This effect is particularly relevant for small values of the SR, while the more we reach higher values of the SR the more we observe an improvement in the prediction
- RoE shows a quite symmetric prediction compared to the actual values, while for SR the overestimation prevails



- Actual vs Predicted analysis to assess if *NN1* can adequately reproduce the actual shape of the combinations of RoE and SR
- Good overall performance, with some areas of lower predictive precision
- Potential improvement of the predictive capability of the model by a specific assessment of the “best” value of each hyper-parameter (e.g. number of nodes, number of hidden layers, etc.)

Neural network approach for selecting efficient reinsurance strategies

Neural network model 2



- Assessment of the efficient frontier produced by the model compared to the observations in the dataset
- In black are reported the combinations of SR and expected RoE for all the reinsurance strategies
- In red is reported the efficient frontier according to *NN2*
- The efficient frontier lies above the set of observations: it means that the model has found a set of strategies that produces a higher Pareto frontier than the one obtained from observed data
- This result is expected since the reinsurance strategies of the observed data are based on a limited number of combinations at a given step-size
- The efficient strategies corresponding to each point of the efficient frontier are derived from the first hidden layer of *NN2*

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Summary of main results and future challenges

- We showed how neural network models can be also applied in the context of reinsurance analysis in the actuarial sector
- These models can be easily employed for finding the efficient frontier in a multi-objective optimization problem, requiring a limited amount of data compared to other approaches and still preserving the possibility of deriving the strategies generating the Pareto front
- The model presented is clearly not the definitive conclusion of the problems we are interested in answering, but it offers a new perspective for its analysis
- Further applications of this neural network model are possible in many different areas of actuarial science
- Another interesting area of research consists in employing explainable methods for deriving information on the characteristics of the efficient strategies derived by the neural network model