FITTING DEVELOPMENT AND TAIL MODELS JOINTLY

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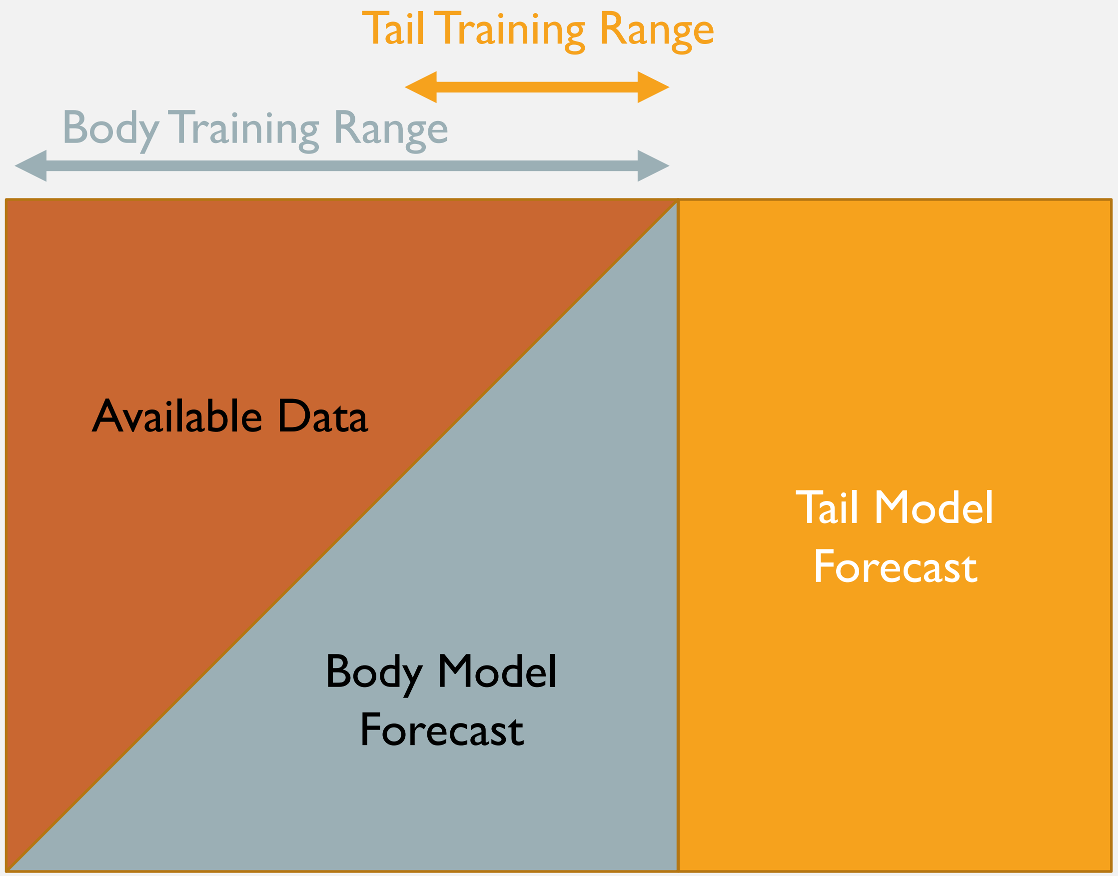
TAIL DEVELOPMENT BACKGROUND

- We wish to estimate the ultimate cost of a pool of casualty insurance claims.
- Claims may take years (or decades) to fully settle
- We may have little or no historical data on fully settled cohorts of claims
- We have to rely on extrapolation to estimate ultimate costs of losses

TAIL DEVELOPMENT BACKGROUND
TRADITIONAL TAIL MODEL SETUP



TRADITIONAL MODEL FUNCTIONAL FORM

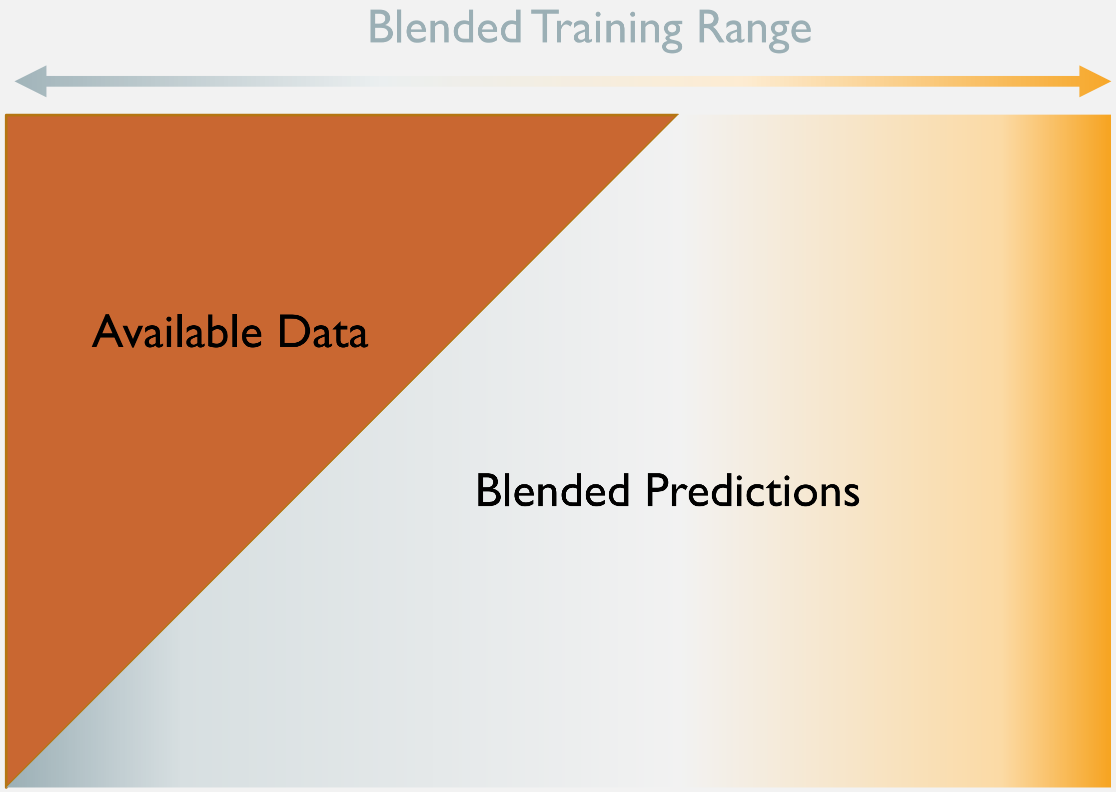
Body Model:

- $\mathbb{E}(L_{t,d}) = L_{t,d-1} ATA_d$
- $Var(L_{t,d}) = \sigma_d^2 L_{t,d-1}$

Tail Model:

- $\mathbb{E}(L_{t,d}) = L_{t,d-1} \exp(\alpha \cdot \gamma^d)$
- $Var(L_{t,d}) = ???$

TAIL DEVELOPMENT BACKGROUND
BLENDED TAIL MODEL SETUP



BLENDED MODEL FUNCTIONAL FORM

Functional Form of Mean:

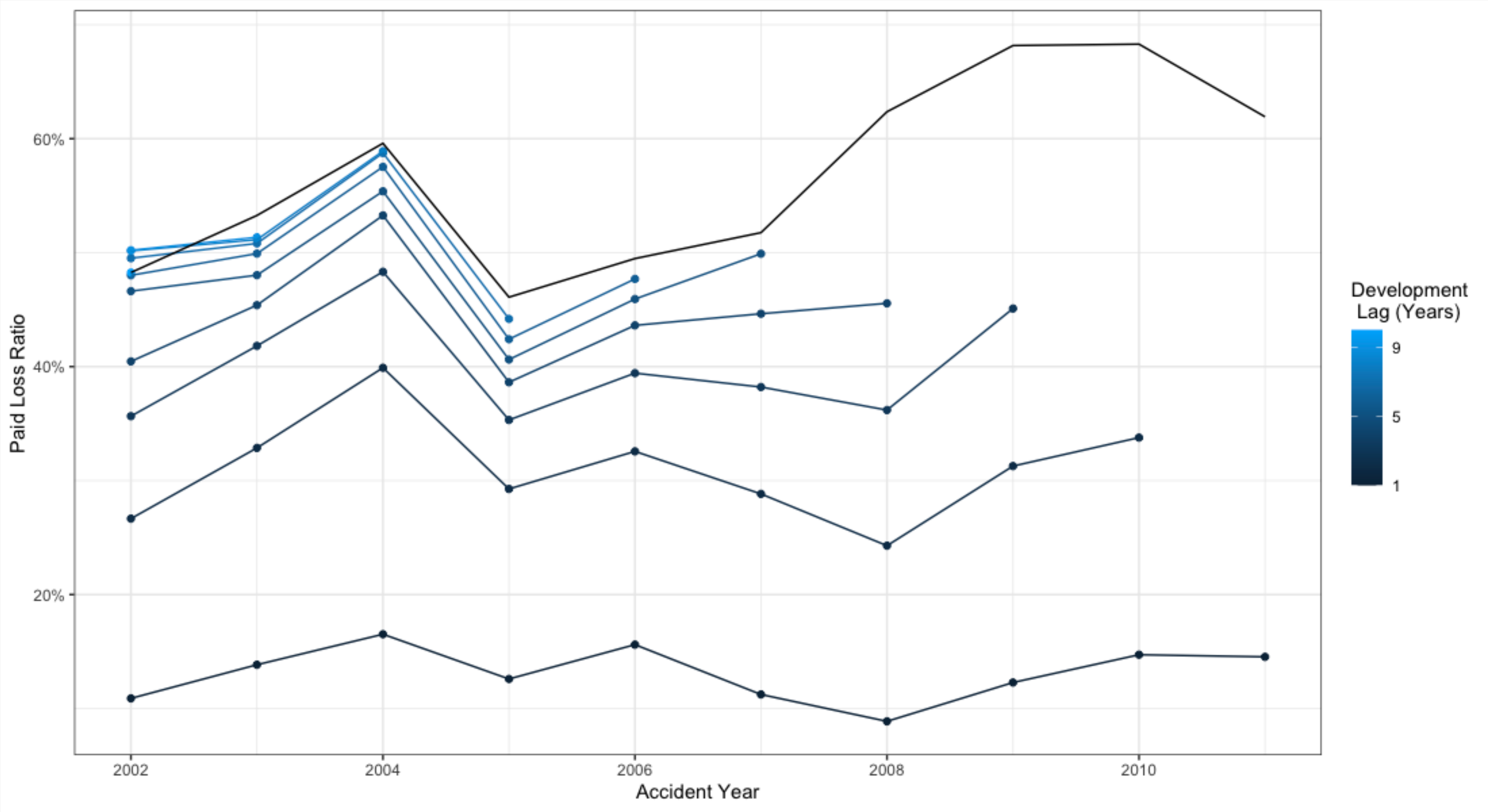
- $\mathbb{E}(L_{t,d}) = L_{t,d-1} ATA_d$
  - $ATA_d = w_{t,d} ATA_{t,d} + \left(1 - w_{t,d}\right) ATA_{b,d}$
  - $ATA_{t,d} = \exp\left(\alpha \cdot \gamma^d\right); \alpha > 0, \gamma > 0$
  - $w_{t,d} = \frac{1}{1+\exp(-\beta_{w,0} - \beta_{w,1} d)}; \beta_{w,1} > 0$

Functional Form of Variance:

- $Var\left(L_{t,d}\right) = \exp\left(\beta_{\sigma,0} + \beta_{\sigma,1} d\right) L_{t,d-1}$

Estimated via Bayesian methods!

BLENDED TAIL DEVELOPMENT COMPARISON



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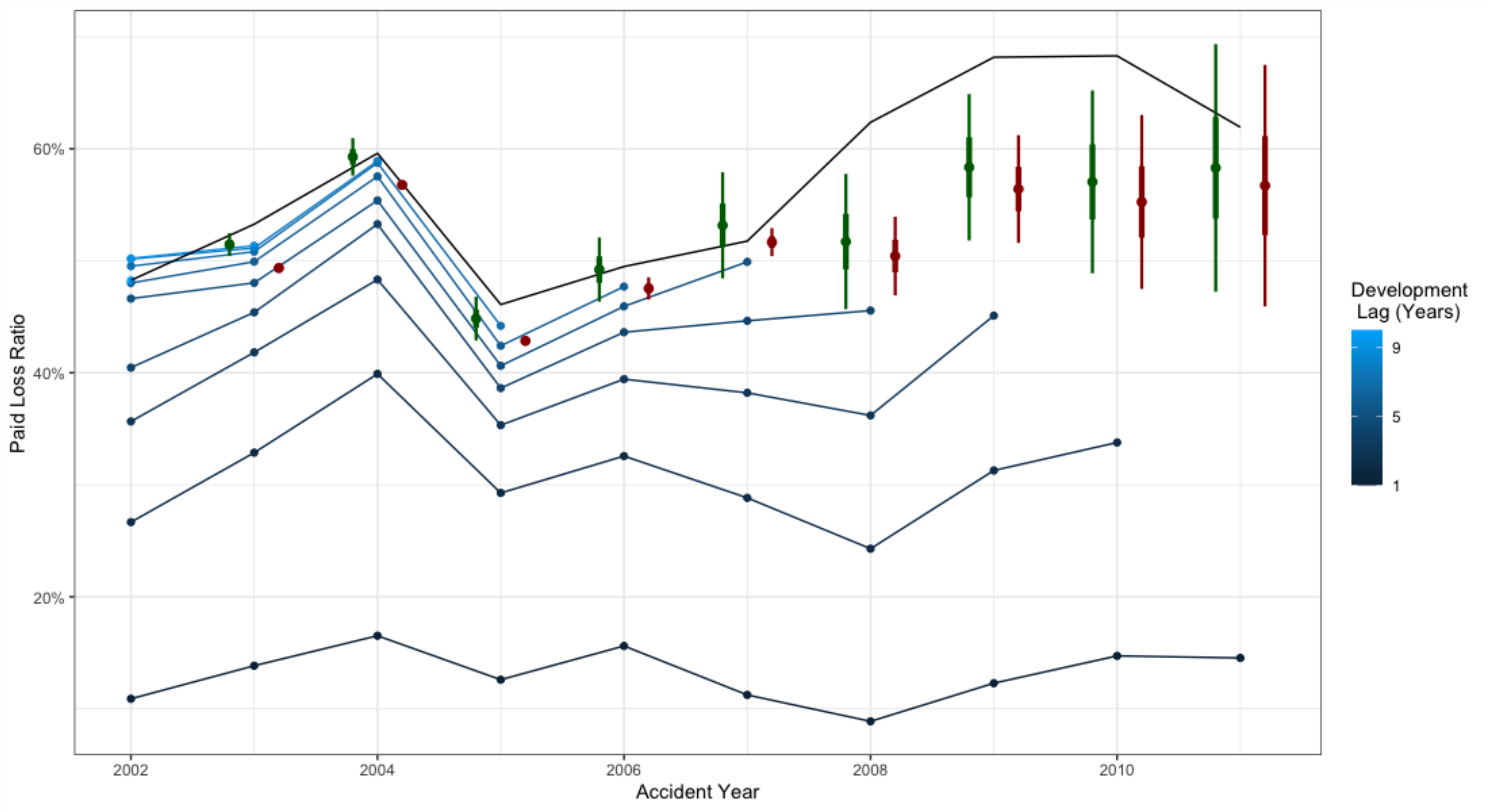

BACKTEST OF BLENDING PERFORMANCE

- Study on 145 10-by-10-year triangles from US statutory filings
  - 35 Commercial Auto
  - 33 General Liability
  - 37 Private Auto Liability
  - 40 Workers' Compensation
- Last accident year in each triangle ranges from 1998 to 2012
- Earned premium in last accident year ranges from $5.3M - $493M
  - IQR of last accident year EP is $23.5M - $148M

BACKTEST OF BLENDING PERFORMANCE

Forecast Interval	Blended MAPE	Mack CL MAPE	Blended Log-Density	MACK CL Log-Density
Lag 9-10	0.65%	0.93%	3.648	-0.001
Lag 8-10	1.17%	1.21%	3.178	0.335
Lag 7-10	1.72%	1.79%	2.642	2.161
Lag 6-10	2.67%	2.84%	2.312	2.105
Lag 5-10	3.43%	3.40%	2.035	1.973
Lag 4-10	5.93%	6.14%	1.540	1.376
Lag 3-10	7.79%	7.81%	1.041	0.949
Lag 2-10	12.01%	12.49%	0.680	0.593
Lag 1-10	14.68%	14.96%	0.436	0.411

BENEFITS OF BLENDED APPROACH

- No judgment required on transition point between body and tail models
- Transition between body and tail is gradual, not abrupt
- Better bias/variance tradeoff on later development lags in body model
- Natural (implicit) weighting of training data in body and tail components
- Shared variance assumptions across body and tail models
- Explicit, easy-to-modify parametric assumptions in the tail

OTHER AREAS OF APPLICATION

- Common situation:
  - Regression on a continuous covariate
  - Expectation of nonlinear relationship in the center of the domain
  - Little data at extreme values of the independent variable
  - Strong intuition about expected behavior at extreme values
  - Estimates at extreme values are important!

OTHER AREAS OF APPLICATION

- Example Applications:
  - Driver age and merit rating factors in private auto insurance
  - Mortality curves in life insurance
  - TIV in inland marine insurance
- Nonparametric methods have implicit assumptions on extrapolation
  - These assumptions may not be plausible
  - Overriding these assumptions may be difficult

QUESTIONS?