Gradient Boosting Machines in Claim Reserve Prediction

Henning Zakrisson
Based on joint research with Mathias Lindholm, Richard Verrall and Felix Wahl

Outline

- The Collective Reserving Model
- Model fitting
- Numerical illustration

The Collective Reserving Model

The Collective Reserving Model (CRM)

Number of claims

$$N_{ij} \sim \mathrm{ODP}\left(\nu_{ij}, \phi\right)$$

for accident year $i = 1, ..., I$ and reporting delay $j = 0, ..., I - 1$

The Collective Reserving Model (CRM)

Number of payments

$$N_{ijk}^{\text{paid}} \mid N_{ij} \sim Po\left(N_{ij}\lambda_{ijk}\right)$$

for payment delay $k = 0, ..., I - 1$

The Collective Reserving Model (CRM)

Aggregated payments

$$X_{ijk} = \sum_{l=1}^{N_{ijk}^{\text{paid}}} Y_{ijkl}$$

The Collective Reserving Model (CRM)

Individual payments i.i.d.

$$\mathbb{E}\left[Y_{ijkl}\right] = \mu_{ijk}$$

$$\mathrm{Var}\left(Y_{ijkl}\right) = \sigma_{ijk}^2$$

The Collective Reserving Model (CRM)

Conditional moments of RBNS payments, uncorrelated by construction

$$\mathbb{E}\left[X_{ijk} \mid N_{ij}\right] = \mu_{ijk}\lambda_{ijk}N_{ij}$$

$$\operatorname{Var}\left(X_{ijk} \mid N_{ij}\right) = \frac{\mu_{ijk}^2 + \sigma_{ijk}^2}{\mu_{ijk}}\mathbb{E}\left[X_{ijk} \mid N_{ij}\right]$$

Model fitting

Model fitting

Assumptions for parameter fitting

$$\mu_{ijk}\lambda_{ijk} = \psi_{ijk}$$

$$\frac{\mu_{ijk}^2 + \sigma_{ijk}^2}{\mu_{ijk}} = \varphi$$

…meaning we can use quasi-Poisson log-likelihood!

Model fitting

(Quasi-)Poisson log-likelihood

$$l\left(\nu_{ij} \mid \mathcal{F}_0\right) = \sum_{i,j:i+j\leq I} \left(N_{ij} \log \nu_{ij} - \nu_{ij}\right)$$

$$l\left(\psi_{ijk} \mid \mathcal{F}_0\right) = \sum_{i,j,k:i+j+k\leq I} \left(X_{ijk} \log\left(N_{ij}\psi_{ijk}\right) - N_{ij}\psi_{ijk}\right)$$

Model fitting

GLM formulation

$$\nu_{ij} = e^{a_i + b_j}$$

$$\psi_{ijk} = e^{\alpha_i + \beta_j + \gamma_k}$$

GBM formulation

$$\nu_{ij} = e^{g(i,j)}$$

$$\psi_{ijk} = e^{h(i,j,k)}$$

Model fitting

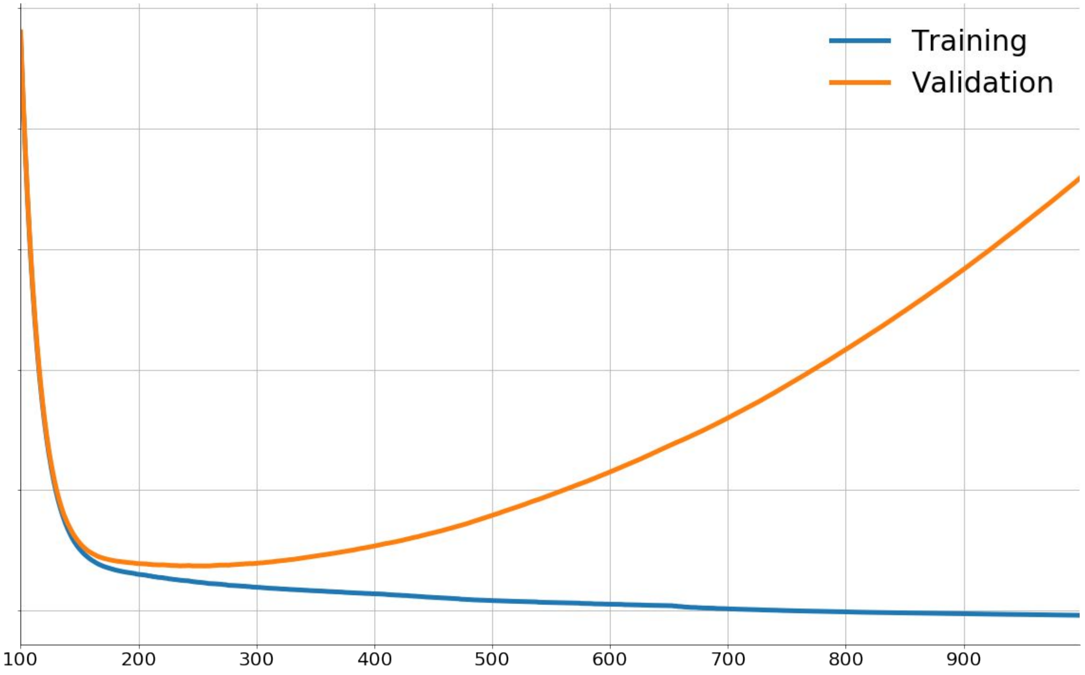
GBM functions

$$g\left(i,j\right)=G_0+\epsilon\sum_{b=1}^{B}r\left(i,j;\gamma_b\right)$$

$$h\left(i,j,k\right)=H_0+\epsilon\sum_{b=1}^{B}r\left(i,j,k;\theta_b\right)$$

Model fitting

Early stopping



Numerical illustration

Numerical illustration

- Data from Gabrielli, Andrea, and Mario V Wüthrich. "An individual claims history simulation machine." *Risks 6.2 (2018)*: 29.
- 6 different LoBs
- I = 12
- Aggregate data to get N_{ij} and X_{ijk}

Numerical illustration

Aggregate the data into two portfolios of equal size

$$\mathcal{N}_0^t = \{N_{ij}^t : i + j \leq I\}$$
$$\mathcal{N}_0^v = \{N_{ij}^v : i + j \leq I\}$$

Such that

$$N_{ij}^v + N_{ij}^t = N_{ij}$$

Numerical illustration

Split the data into two portfolios of equal size

$$\mathcal{X}_0^t = \{X_{ijk}^t : i + j + k \leq I\}$$
$$\mathcal{X}_0^v = \{X_{ijk}^v : i + j + k \leq I\}$$

Such that

$$X_{ijk}^t + X_{ijk}^v = X_{ijk}$$

Numerical illustration

Predict reserves

$$\hat{R} = \sum_{i,j,k:i+j+k>I} \mathbb{E}_{\hat{\psi},\hat{\nu}}\left[X_{ijk} \mid \mathcal{N}_0\right]$$

$$\mathbb{E}_{\hat{\psi},\hat{\nu}}\left[X_{ijk} \mid \mathcal{N}_0\right] = \mathbb{E}_{\hat{\nu}}\left[N_{ij} \mid \mathcal{N}_0\right] \hat{\psi}_{ijk}$$

$$\mathbb{E}_{\hat{\nu}}\left[N_{ij} \mid \mathcal{N}_0\right] = \begin{cases} N_{ij}, & i + j \leq I \\ \nu_{ij}, & i + j > I \end{cases}$$

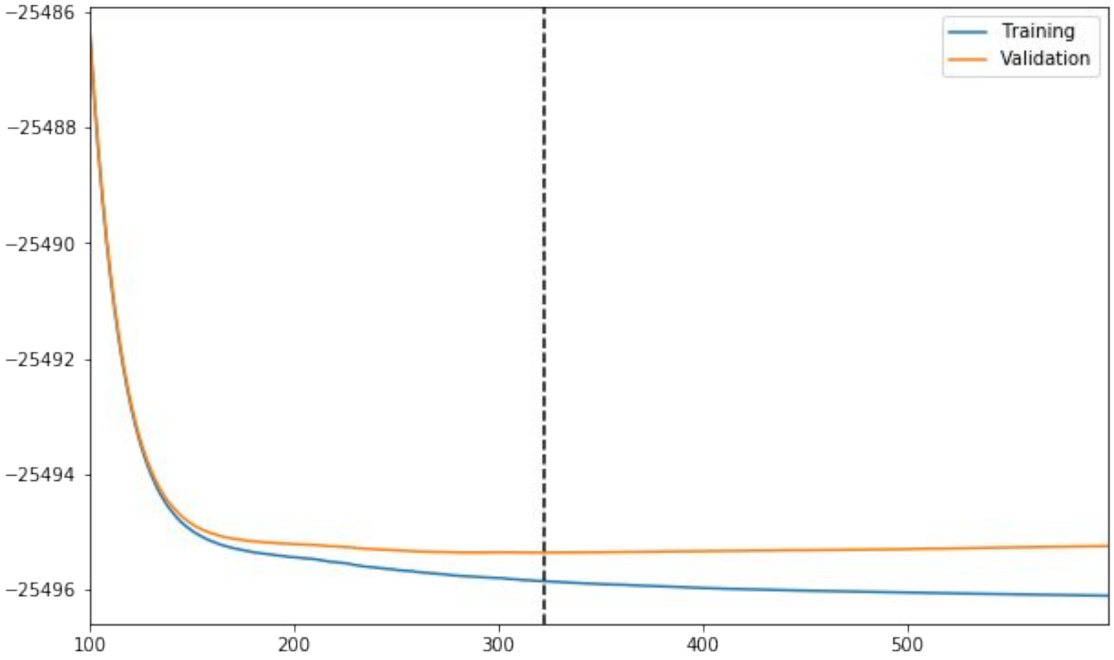
Numerical illustration

Number of claims training, LoB 1

Max depth: 2

Step size: 0.1

Number of trees: 322



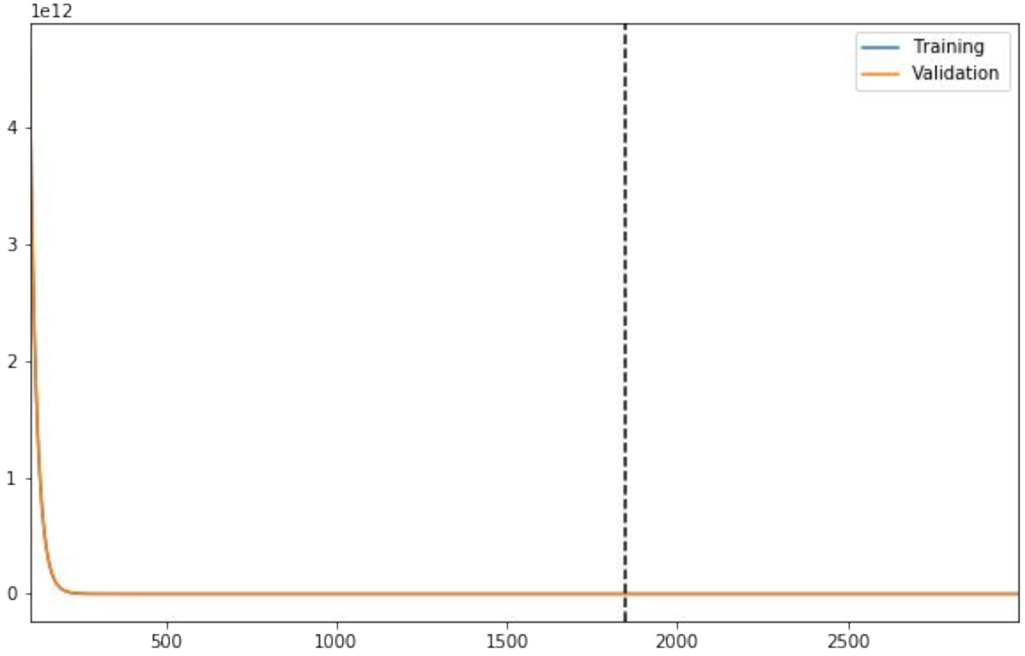
Numerical illustration

Aggregated payment training, LoB 1

Max depth: 1

Step size: 0.1

Number of trees: 1847



Numerical illustration

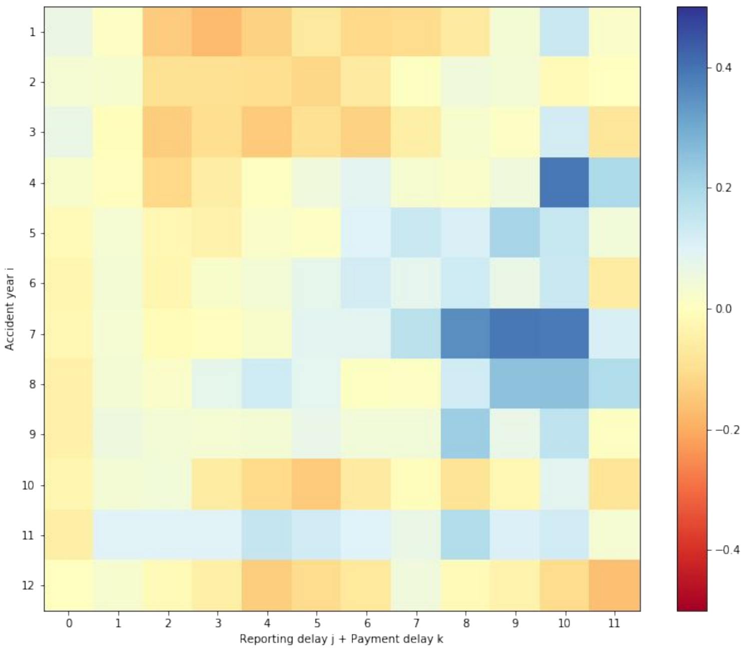
Reserve residuals

	LoB 1	LoB 2	LoB 3	LoB 4	LoB 5	LoB 6
CL	-2.82%	-4.26%	-7.02%	-5.66%	-3.28%	-5.49%
GLM	-3.45%	-5.12%	-8.42%	-6.40%	-4.23%	-6.25%
GBM	+2.17%	+0.03%	-5.17%	-1.75%	-0.76%	-1.29%

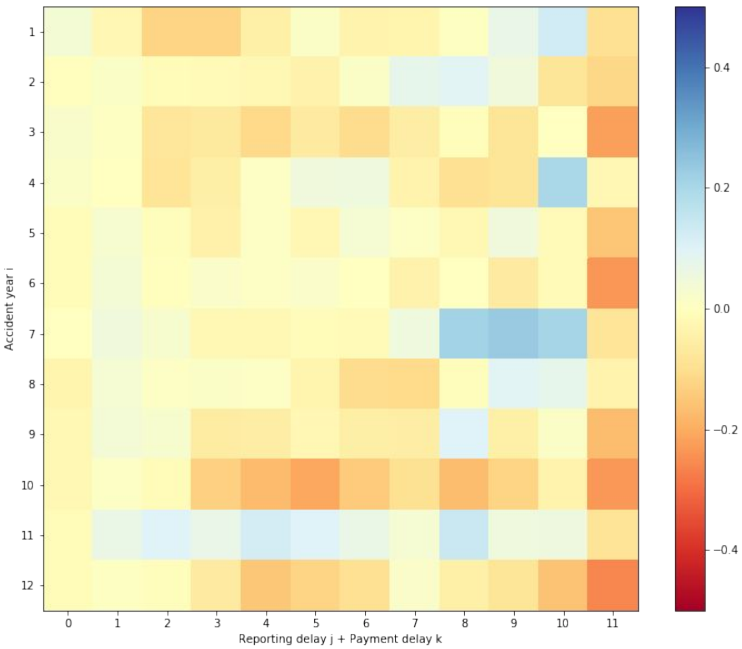
Numerical illustration

Residual heatmaps, LoB 1

GLM

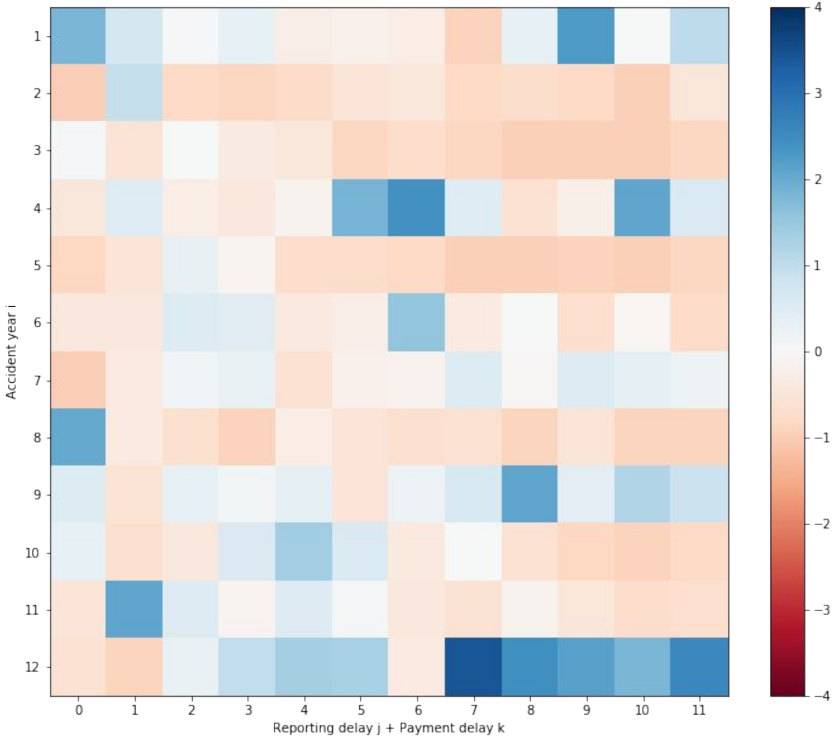


GBM



Numerical illustration

Aggregated payment (over)overdispersion $\varphi_{i,j+k}/\varphi - 1$, LoB 1



Thank you for listening

References

Lindholm, Mathias, et al. "Machine Learning, Regression Models, and Prediction of Claims Reserves." *Casualty Actuarial Society E-Forum Summer 2020*.

Friedman, Jerome H. "Greedy function approximation: a gradient boosting machine." *Annals of statistics (2001)*: 1189-1232.

Wahl, Felix, Mathias Lindholm, and Richard Verrall. "The collective reserving model." *Insurance: Mathematics and Economics 87 (2019)*: 34-50.

Gabrielli, Andrea, and Mario V Wüthrich. "An individual claims history simulation machine." *Risks 6.2 (2018)*: 29.