Metamodeling for Variable Annuity Valuation: What works and what does not

Xiaochen Jing

University of Wisconsin - Madison

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- Variable Annuities (VAs) are separate account life insurance contracts linked to a list of financial instruments with tax benefits
 - Equipped with various optional features
- Metamodeling for VA Portfolio Valuation: a very active research topic in actuarial science
 - Gan (2015), Gan & Lin (2017), Wu et al. (2018), Quan et al. (2021),...
 - All papers so far rely on synthetic datasets
- Research Question: How well do metamodeling approaches work on real-world VA contracts?
- Extract contract features and build a data set of VAs with GMABs
 - Implement a flexible MC simulation process for VA valuation
- Test Metamodeling with different sampling and learning components
 - Larger sample size v
 - Sophisticated learners
 - Sampling methods ×

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- VA prospectus: Typically several hundred-page long documents with detailed description
 - Gan and Valdez (2017): "...extremely difficult, if not impossible, for researchers to obtain real datasets..."

- Our source: Morningstar Annuity Intelligence
 - 2,346 VA + GMAB combinations (22,623 in total for all benefit types)
 - From 1994 to 2017
 - Numerical values on fees and benefits
 - Textual description on features and conditions

Page 3 Real Variable Annuity Contract Data

Morningstar Report Example

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M&E Fee: The M&E is based on the Return of Premium Death Benefit. <u>The optional Standard Death</u> <u>Benefit is available for 0.15% less</u>. Annual Policy Fee: The annual policy fee is \$30 for contracts issued in NM. Prior to 7/18/2011, the annual policy fee is \$35 (\$30 for contracts issued in ND) and is waived if the anniversary contract value is at least \$50,000. Prior to 7/18/2011, the surrender schedule is 7,6,6,5,4,3,2 and the free withdrawal amount is the greater of 10% of adjusted purchase payments (must be systematic for first year) or all earnings.

Example of Feature Extraction

Feature Description	Textual Information	Data Extraction			
Benefit Charge	0.750% assessed annually	fee_B_Base = AV;			
Denent Gharge	and calculated against the account value	fee_B_Rate = 0.0075			
Surrender Charge Schedule	764	Length = 3; Slope = -1.5;			
Surrender Sharge Schedule	7, 0, 4	Max = 7; Min = 4			
Impact of Withdrawal	Proportionate	IW = Proportionate			
	On the 5th or subsequent anniversary the				
	benefit base can be stepped-up to the current	SU_Base = reset; SU_Rate = 1;			
Step-up Option	account value. Electing a step-up will restart	Initial = 5; Frequency = 1;			
	the ten-year waiting period. Future step-ups	Waiting = 10; Next = 5;			
	are available five years after the last step-up.				
	(Benefit) Fee percentage is 0.55% prior to	If ID < 11/1/2010: fee_B_Rate == 0.0055;			
Specialities	11/1/2010, 0.40% prior to 3/2/2009 and	If ID < 3/2/2009: fee_B_Rate == 0.0040;			
	0.25% prior to 5/1/2008.	If ID < 5/1/2008: fee_B_Rate == 0.0025;			

- 53 contract features for valuation and learning
- Black-Scholes framework with 648 scenarios for each contract

Page 5 Metamodeling on VA valuation Sample Selection and Statistical Learning

Select \Rightarrow Calculate \Rightarrow Learn \Rightarrow Predict

- Representative Sample Selection
 - Random Sampling
 - Clustering (k-means)
 - Latin Hypercube Sampling



- Statistical Learning
 - Generalized Linear Model (baseline)
 - Tree-Based Models
 - Neural Network

Page 6 Metamodeling on VA valuation Results on Sample Selection

- Similar distribution
- Time consuming vector quantization

	Random Sample			Latin I	Hypercube	Cluster Sampling		
 Sample Size	1%	5%	20%	1%	5%	1%	5%	
Sampling Time (h)	-	-	-	3.42	20.51	3.54	24.74	





Page 7 Metamodeling on VA valuation Important Features (Standardized)

- GLM picks up dummies for feature categories
- Boosted Trees emphasize on fees

GLM		Boosted Trees				
Feature Name	Coefficient	Feature Name	Importance			
IW Speciality	10.0609	BenefitFee	11.07%			
StepUp Next	8.6718	SubAccountFee L	8.69%			
StepUp Initial	-8.3399	VA Fee	6.98%			
IW (min)	-5.0211	M&E Fee	6.16%			
IW (dollar)	-3.4742	SubAccountFee U	6.06%			
FreeWithdrawal Base (AV)	-3.182	WithdrawalStrategy 1	5.12%			
FreeWithdrawal Base (BB)	-2.8338	SurranderCharge Slope	4.65%			
BenefitFee Speciality	2.6186	Age	3.97%			
WithdrawalStrategy 3	-2.4723	AnnuitizationAge	3.87%			
BenefitFee Base (max)	2.4433	WithdrawalStrategy 3	3.75%			

Page 8 Metamodeling on VA valuation

Accuracy and Runtime of Metamodeling

		Random Sample			Latin Hypercube		Cluster Sampling	
	Sample Size	1%	5%	20%	1%	5%	1%	5%
GLM	Tuning Time (h)	-	-	-	-	-	-	-
	OOS RMSE	4.29	4.29	4.27	4.29	4.29	4.30	4.30
Boosted Trees	Tuning Time (h)	1.79	22.17	192.29	2.52	21.43	2.15	21.15
	OOS RMSE	3.77	3.15	3.04	3.78	3.20	3.77	3.21
Random Forest	Tuning Time (h)	0.08	0.43	2.70	0.08	0.44	0.13	0.56
	OOS RMSE	3.69	3.13	2.65	3.70	3.16	3.72	3.14
Neural Network	Tuning Time (h)	7.40	34.15	193.56	7.80	30.50	6.26	25.68
	OOS RMSE	4.00	3.53	3.45	4.02	3.53	4.13	3.55

• GLM isn't improving with more samples.

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- Tuning time scales heavily with sample sizes
- RMSE decreases for about 20% with a 20× increase in sample size

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- RMSE of \$2-\$3, with mean of the actual value around \$13 (s.d. of \$5)
- MAPE of 20%, PE on the portfolio level < 0.1%

Thank you!

Metamodeling for Variable Annuity Valuation

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