Risk Budgeting Portfolios from Simulations

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16/Jun/2021

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Introduction

- An investment concept used in industry, e.g., by pension funds, is **risk budgeting portfolios (RBP)**
  - **RBP**: diversified portfolios on the level of the **risk contribution** of each asset
  - **risk contribution**: the added risk of marginally increasing the portfolio's position in that asset
- Our contribution: to use **stochastic optimization** to construct RBPs based on a generic **risk measure** and **simulations** from the returns.

The setup

- At time $t = 0$:
  - Assets prices: $\boldsymbol{p} = (p_1, \ldots, p_d)$
  - The investor holds: a_i shares of asset i
  - Portfolio value: $v(\boldsymbol{a}) = \boldsymbol{a}^\intercal \boldsymbol{p}$
- At time $t = 1$:
  - Assets prices: $\boldsymbol{P} = (P_1, \ldots, P_d)$
  - The investor holds: a_i shares of asset i
  - Portfolio value: $V(\boldsymbol{a}) = \boldsymbol{a}^\intercal \boldsymbol{P}$
- **Portfolio loss**: $L(\boldsymbol{a}) = -[V(\boldsymbol{a}) - v(\boldsymbol{a})] = \sum_{i=1}^{d} a_i L_i$
  - i-th asset loss: $L_i = -(P_i - p_i)$
- **Initial endowment**: v_0 (dollars) at time $t = 0$.

Marginal Risk and Risk Contribution

- **Marginal risk** of asset i (with coherent risk measure ρ)

$$\mathcal{MR}_i(\boldsymbol{a}) = \frac{\partial}{\partial a_i}\rho(\boldsymbol{a}^\intercal \boldsymbol{L})$$

- **Risk contribution** of asset i

$$\mathcal{RC}_i(\boldsymbol{a}) = a_i\,\mathcal{MR}_i(\boldsymbol{a})\,.$$

- For a portfolio $\boldsymbol{a}$ and a risk measure ρ, it holds

$$\rho(L(\boldsymbol{a})) = \sum_{i=1}^{d} a_i\,\mathcal{MR}_i(\boldsymbol{a}) = \sum_{i=1}^{d}\mathcal{RC}_i(\boldsymbol{a})\,.$$

The Risk Budgeting Portfolio

- The investor has a **risk appetite** $B^\dagger$ split as $B^\dagger = \sum_{i=1}^d B_i$
- For a **risk budget** $\boldsymbol{B} = (B_1, \ldots, B_d)$ the RB portfolio satisfies

$$B^\dagger = \rho(L(\boldsymbol{a})) \ \text{ and } \ B_i \, \mathcal{RC}_j(\boldsymbol{a}) = B_j \, \mathcal{RC}_i(\boldsymbol{a})$$

Proposition (RB portfolio)

The RB portfolio's weights are the solution to

$$\boldsymbol{a}^* := argmin_{\boldsymbol{a}} \, \rho(L(\boldsymbol{a})) \,, \ \textit{subject to} \quad \sum_{i=1}^{d} B_i \log(a_i) \geq 0,$$

where the a_i are implicitly constraint to be strictly positive, so that the log is well-defined.

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- With **simulated** returns' scenarios, we can estimate

$$\hat{\rho}(L(\boldsymbol{a})) \approx \rho(L(\boldsymbol{a}))$$

and use the Sample Average Approximation method

Proposition (SAA-RB portfolio)

The SAA-RB portfolio's weights are the solution to

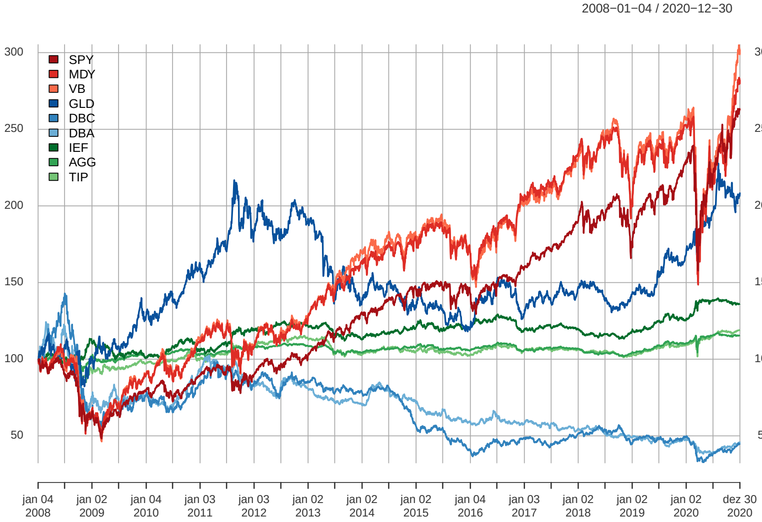
$$\boldsymbol{a}^* := \mathit{argmin}_{\boldsymbol{a}} \, \hat{\rho}(L(\boldsymbol{a})) \, , \text{ subject to } \quad \sum_{i=1}^{d} B_i \log(a_i) \geq 0,$$

where the a_i are implicitly constraint to be strictly positive, so that the log is well-defined.

Example: Multi-class RB portfolio

Ticker	Name	Asset class	RB weight
SPY	SPDR S&P 500 ETF	Equities	2
MDY	SPDR S&P MidCap 400 ETF	Equities	1
VB	Vanguard Small-Cap ETF	Equities	1
GLD	SPDR Gold Shares	Commodities	2
DBC	PowerShares DB Comm. Tracking ETF	Commodities	1
DBA	PowerShares DB Agriculture ETF	Commodities	1
IEF	iShares 7–10 year Treasury Bond	Bonds	2
AGG	iShares Core U.S. Aggregate Bond	Bonds	1
TIP	iShares TIPS Bond	Bonds	1

Example: Multi-class RB portfolio



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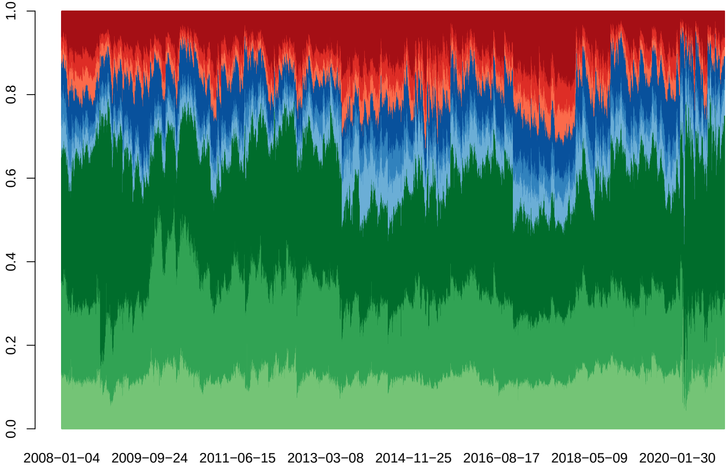

Figure: SPY, MDY, VB, GLD, DBC, DBA, IEF, AGG, TIP

Example: Multi-class RB portfolio

	Vol.	Return	Sharpe	$VaR_{5\%}$	Drawdown
grp	5.72%	3.12%	**0.545**	-0.53%	-17.26%
gmv	5.96%	2.68%	0.450	-0.47%	-18.17%
ew	11.07%	**5.11%**	0.462	-1.00%	-30.58%
rp095	**5.06%**	2.71%	0.536	**-0.47%**	**-15.86%**