Compartmental Reserving

a new reserving approach implemented in R

Jake Morris

29 June 2015

Agenda

- Background
- Methodology
- Implementation in R
- Future development
- Conclusions

Background

Motivation

- Observed phenomena are governed by underlying processes, with varying degrees of complexity

- A modeller often has to balance:
  - Depth/detail
  - Parsimony

  Trade-off

- However, many common claims reserving methods do not:
  - Explicitly consider the claims process
  - Build model complexity from the "ground up"

Risk of mistaking noise for signal

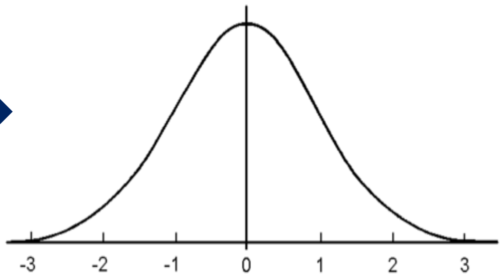
Background

Parsimony

Mixed-effects ("hierarchical") modelling

Cohorts

Parameters a "mixture" of those varying across cohort and those not*



<table>
<tr><th>Cohort</th><th>P_1</th><th>P_2</th><th>P_3</th><th>P_4</th></tr>
<tr><td>1</td><td>$P_{1,1}$</td><td rowspan="4">P_2</td><td>$P_{3,1}$</td><td rowspan="4">P_4</td></tr>
<tr><td>2</td><td>$P_{1,2}$</td><td>$P_{3,2}$</td></tr>
<tr><td>3</td><td>$P_{1,3}$</td><td>$P_{3,3}$</td></tr>
<tr><td>4</td><td>$P_{1,4}$</td><td>$P_{3,4}$</td></tr>
</table>

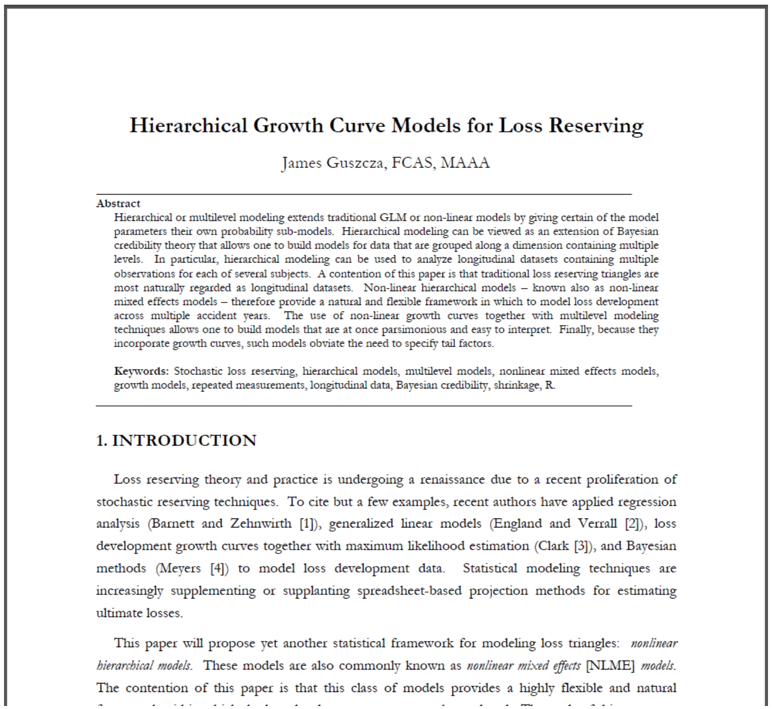
Only estimate mean and s.d. of the variable parameters

*Also known as a mixture of "random effects" and "fixed effects"

Background

Loss reserving

- In 2008, Guszcza showed us how to apply nonlinear mixed effects models to loss reserving*:



Hierarchical Growth Curve Models for Loss Reserving

James Guszcza, FCAS, MAAA

Abstract

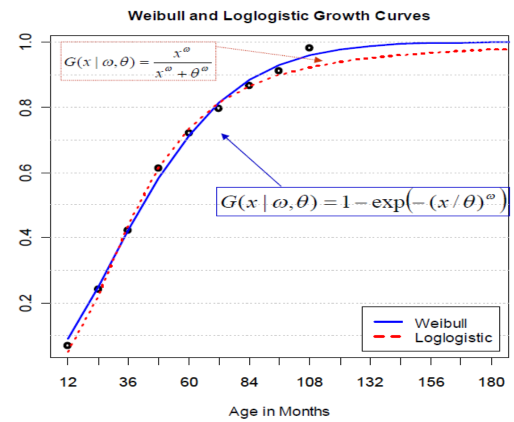
Hierarchical or multilevel modeling extends traditional GLM or non-linear models by giving certain of the model parameters their own probability sub-models. Hierarchical modeling can be viewed as an extension of Bayesian credibility theory that allows one to build models for data that are grouped along a dimension containing multiple levels. In particular, hierarchical modeling can be used to analyze longitudinal datasets containing multiple observations for each of several subjects. A contention of this paper is that traditional loss reserving triangles are most naturally regarded as longitudinal datasets. Non-linear hierarchical models – known also as non-linear mixed effects models – therefore provide a natural and flexible framework in which to model loss development across multiple accident years. The use of non-linear growth curves together with multilevel modeling techniques allows one to build models that are at once parsimonious and easy to interpret. Finally, because they incorporate growth curves, such models obviate the need to specify tail factors.

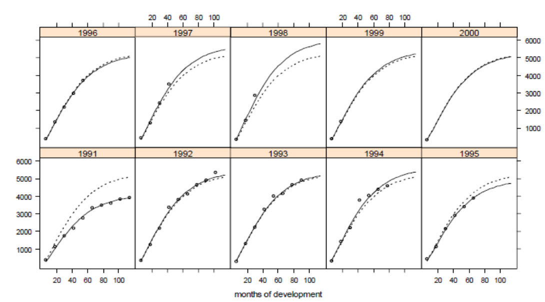
Keywords: Stochastic loss reserving, hierarchical models, multilevel models, nonlinear mixed effects models, growth models, repeated measurements, longitudinal data, Bayesian credibility, shrinkage, R.

1. INTRODUCTION

Loss reserving theory and practice is undergoing a renaissance due to a recent proliferation of stochastic reserving techniques. To cite but a few examples, recent authors have applied regression analysis (Barnett and Zehnwirth [1]), generalized linear models (England and Verrall [2]), loss development growth curves together with maximum likelihood estimation (Clark [3]), and Bayesian methods (Meyers [4]) to model loss development data. Statistical modeling techniques are increasingly supplementing or supplanting spreadsheet-based projection methods for estimating ultimate losses.

This paper will propose yet another statistical framework for modeling loss triangles: *nonlinear hierarchical models*. These models are also commonly known as *nonlinear mixed effects* [NLME] *models*. The contention of this paper is that this class of models provides a highly flexible and natural





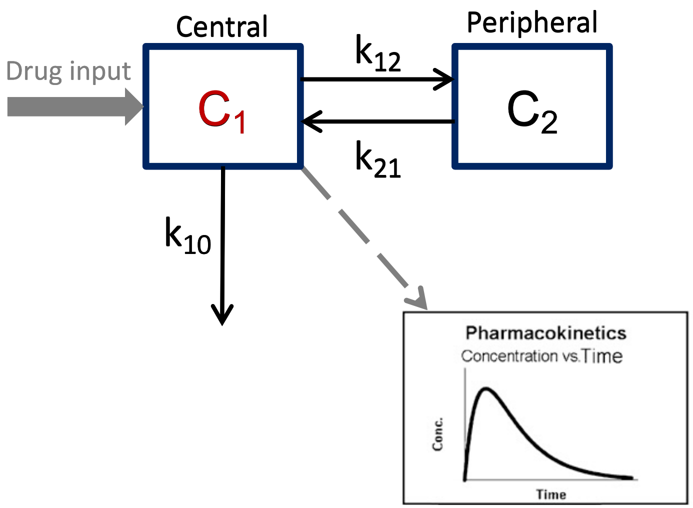
*__Key idea__: fit a nonlinear parametric curve to cumulative paid triangles in a mixed effects modelling framework

Background

Drug development

- Hierarchical models are used routinely in the pharmaceutical industry:

"Compartmental" Pharmacokinetic models



Meaningful parameters and extensibility

Can we apply this modelling framework to loss reserving?

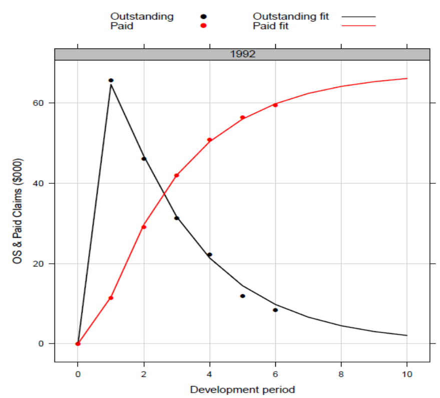
Methodology

Structural model

Compartmental model

- Claim “flows” between compartments governed by ODEs*

- Fit to outstanding and paid triangles
  - Viewed together
  - Simultaneously, capturing tails



*ODEs: a collection of simultaneous Ordinary Differential Equations

Methodology

Parameters

Parameters have natural interpretations

Reported loss ratio ("**RLR**")

Rate of earning + reporting ("$\mathbf{k_{er}}$")

Reserve robustness factor ("**RRF**")

Rate of payment ("$\mathbf{k_p}$")

ULR = RLR*RRF

Rates can optionally vary with development time

Base model parameters for a single cohort

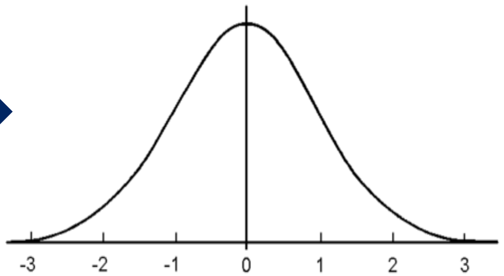
Methodology

Statistical framework

Mixed-effects modelling

Cohorts

Parameters a "mixture" of those varying across cohort and those not*



Cohort	RLR	k_{er}	RRF	k_p
1	RLR_1	k_{er}	RRF_1	k_p
2	RLR_2		RRF_2	
3	RLR_3		RRF_3	
4	RLR_4		RRF_4	

Only estimate mean and s.d. of the variable parameters

*Also known as a mixture of "random effects" and "fixed effects"

Methodology

Data requirements (1)

Minimum data requirements

- Cumulative triangles

*Provided approx. all premiums are written by Time 0

Methodology

Data requirements (2)

Maximum data requirements

- Cumulative triangles

*Required for UW year data

Implementation in R

Why R?

- **Nonlinear mixed effects models require complex solver algorithms:**

Response y {OS,PD}
=
Non-linear function f of
(Parameter vector ϕ and time t)
+
Noise w

$$L(\underline{\beta},\underline{\eta},\sigma \mid \Im_0^{(\omega_0)}) = \prod_{i\in I}\prod_{c\in C}\int_{\underline{b}^{(i)}\in\Re^{SizeP}} pdf(\underline{y}^{(i,c)}(\omega_0)\mid \underline{b}^{(i)},\underline{\beta},\sigma)\cdot pdf(\underline{b}^{(i)}\mid\underline{\eta},\sigma)\cdot d\underline{b}^{(i)}$$

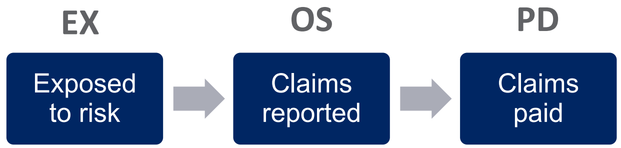
We don't have to worry about this!

- **"f" is derived by solving ODEs:**

$$dEX/dt = -k_r \cdot EX$$

$$dOS/dt = k_{er} \cdot RLR \cdot EX - k_p \cdot OS$$

$$dPD/dt = k_p \cdot RRF \cdot OS$$



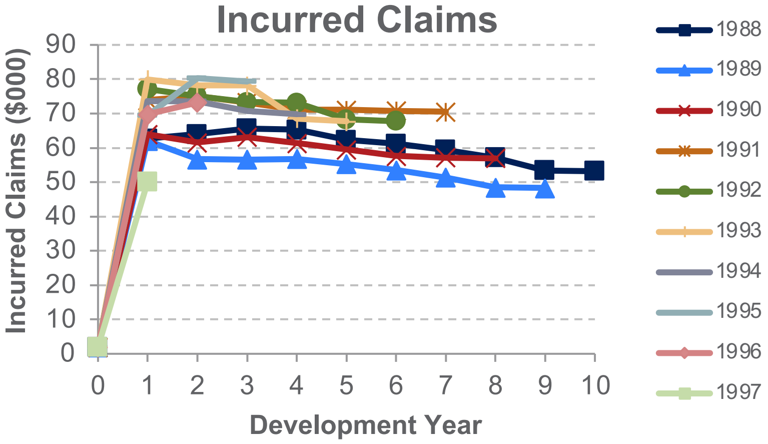
R packages "nlmeODE" and "nlme" do the work*

*Other software packages also usable

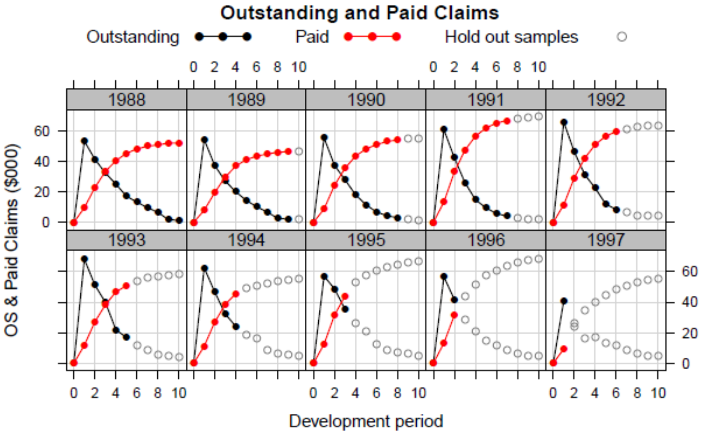
Implementation in R

Case study

- **Workers' Comp Schedule P data**
  - Accident year cohorts (1988 – 1997)
  - Earned premiums
  - Paid and Incurred claims development



- **Aims**
  - Fit compartmental model to data
    - Improve model as necessary
  - Extrapolate to time 10 and ultimate
  - Compare results to hold out samples



Implementation in R

Model 1

```
#####################################
###DEFINE BASE COMPARTMENTAL MODEL###
#####################################

DEmodel <- list(
                DiffEq=list(
                    dy1dt = ~ -ker*y1,
                    dy2dt = ~ ker*RLR*y1 - kp*y2,
                    dy3dt = ~ kp*RRF*y2),
                ObsEq=list(
                   EX   = ~ 0,
                   OS   = ~ y2,
                   PA   = ~ y3),
States=c("y1","y2","y3"),
Parms=c("ker","RLR","kp","RRF"),
Init=list(0,0,0))

ReservingModel <- nlmeODE(DEmodel,Data)

#########################
###FIT BASE NLME MODEL###     Random effects for RLR and RRF
#########################

nlmeModel <- nlme(Claims ~ ReservingModel(ker,RLR,kp,RRF,t,Cohort,Type),
data = Data,
fixed = ker+RLR+kp+RRF ~ 1,
random = pdDiag(RLR + RRF ~ 1),
groups = ~Cohort,
weights = varIdent(form = ~1 | Type),
start=c(ker = log(1.5), RLR = log(1), kp = log(0.75), RRF = log(0.75)),*
control=list(returnObject=TRUE,msVerbose=TRUE,
msMaxIter=10000,pnlsMaxIter=10000,
pnlsTol=0.4,
verbose=TRUE))
```

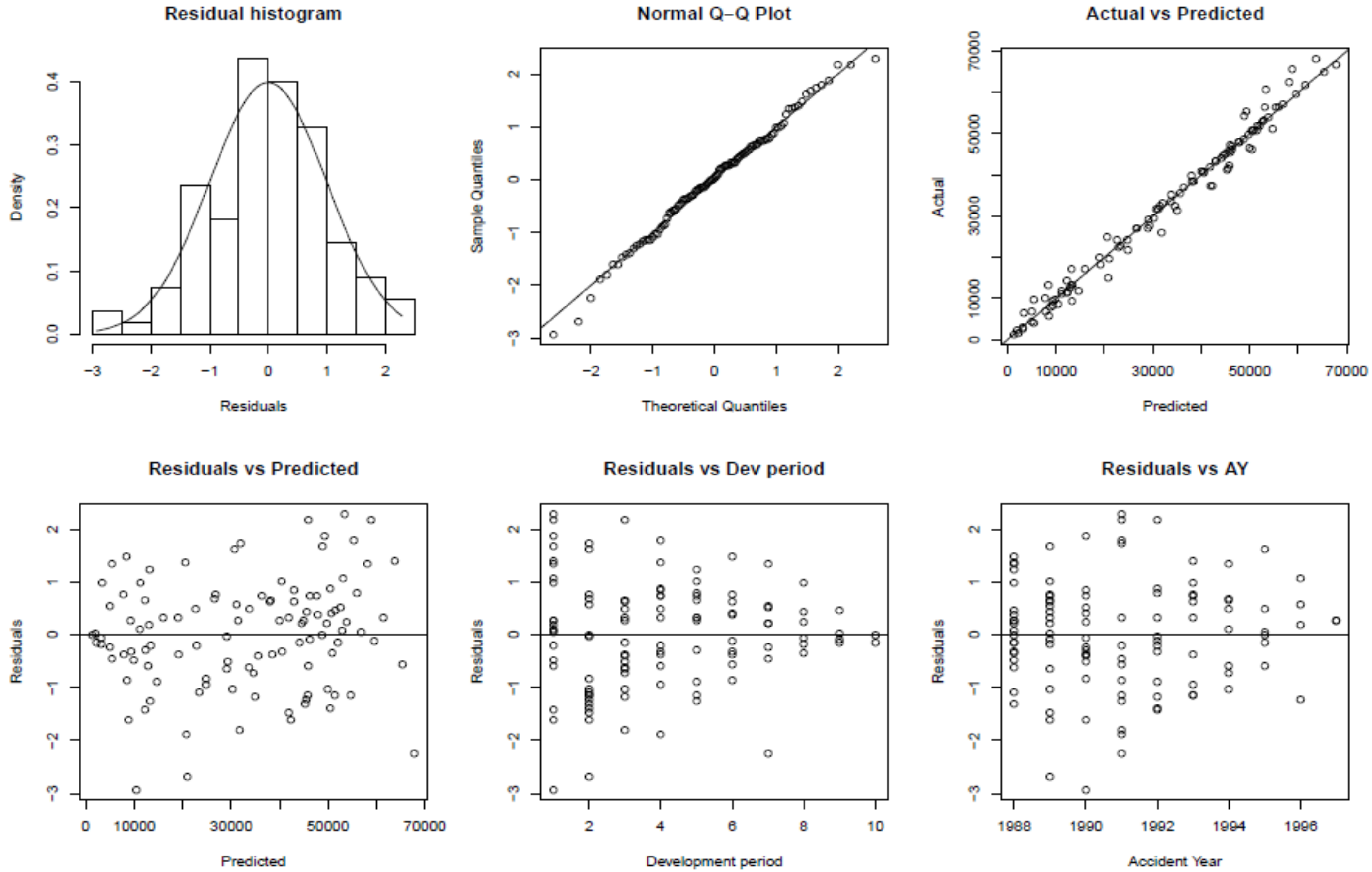
Cohort	RLR	k_{er}	RRF	k_p
1	RLR_1	k_{er}	RRF_1	k_p
2	RLR_2		RRF_2	
3	RLR_3		RRF_3	
4	RLR_4		RRF_4	

Convergence time: 2.5 seconds

Estimate ln(parameters) s.t. cannot be < 0; **Parameters therefore assumed to be lognormal**

Implementation in R

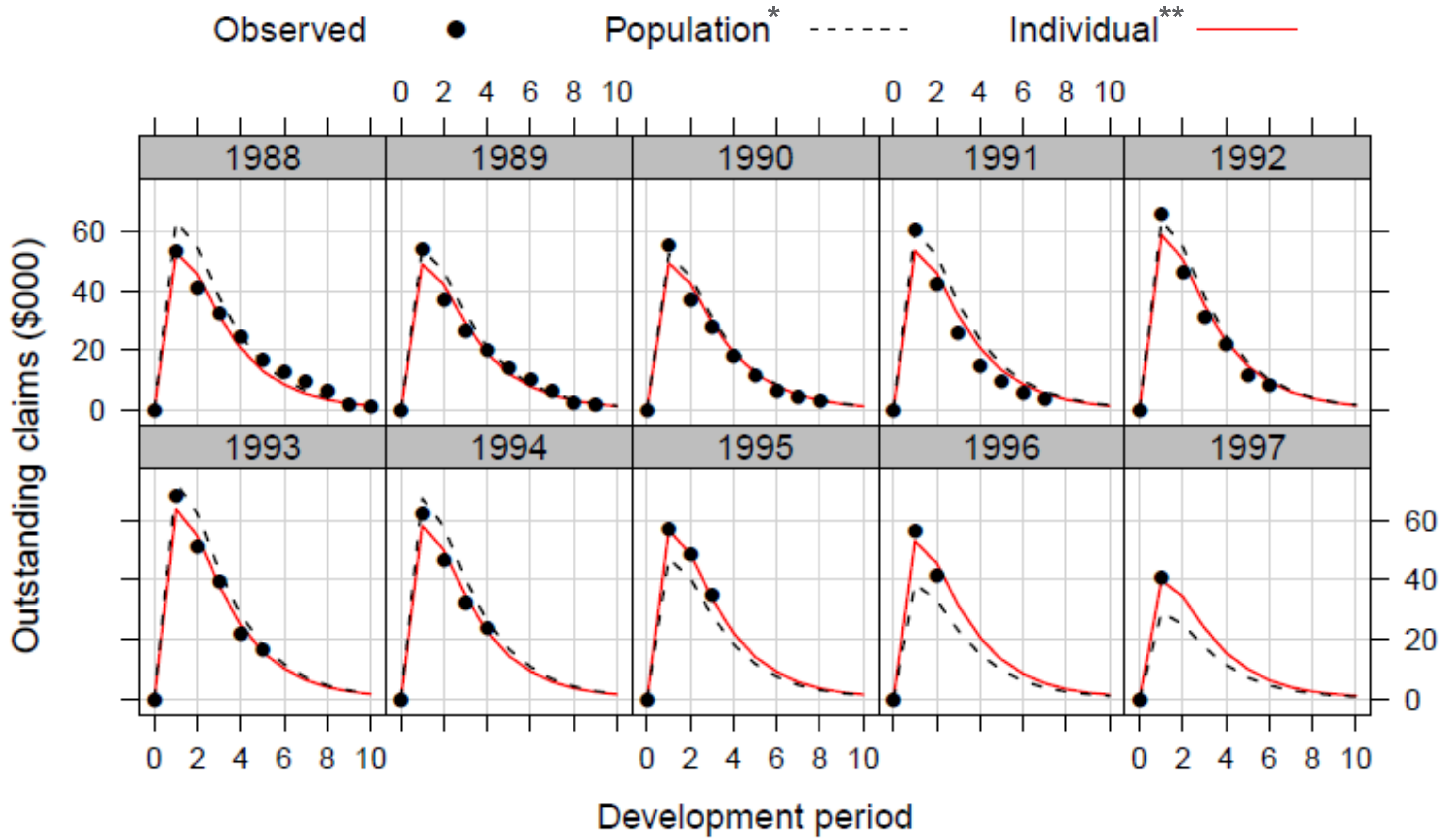
Model 1 Diagnostics



May consider “Jarque-Bera” and “Shapiro-Wilks” tests of residual normality

Implementation in R

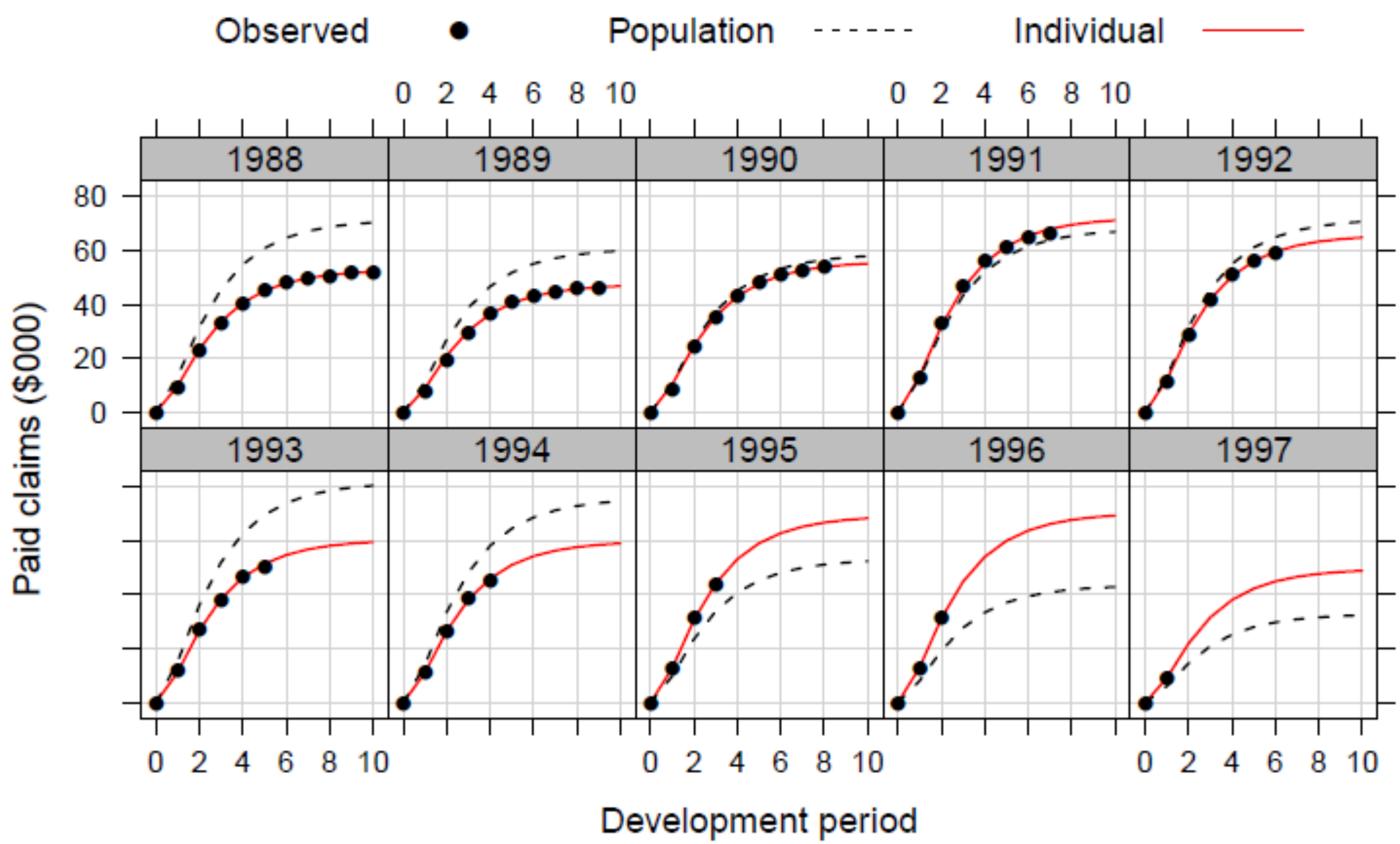
Model 1 O/S fits



***Population:** model fit *not* allowing parameters to vary by cohort
****Individual:** model fit allowing parameters to vary by cohort

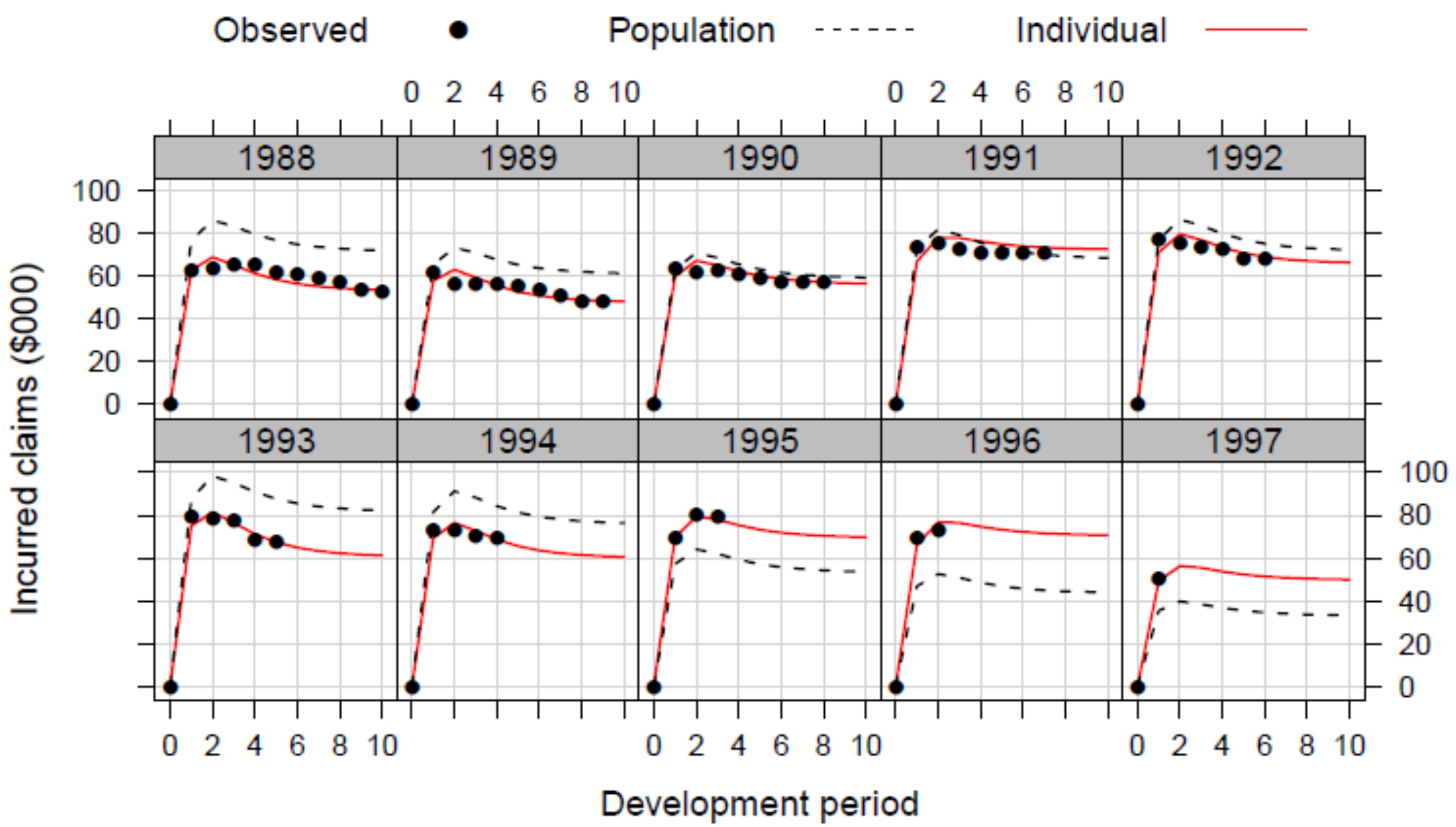
Implementation in R

Model 1 paid fits



Implementation in R

Model 1 incurred fits



Implementation in R

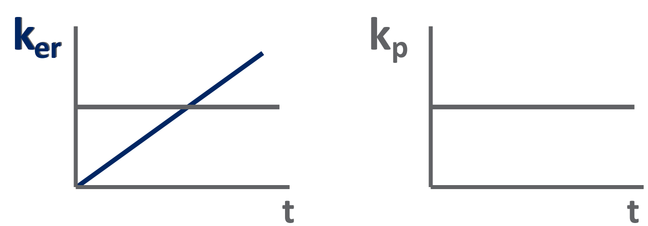
Model 2

```
####################################
###DEFINE ALT. COMPARTMENTAL MODEL### Rate of Reporting increase with dev time
####################################

DEmodel <- list(
                DiffEq=list(
                    dy1dt = ~ -ker*t*y1,
                    dy2dt = ~ ker*t*RLR*y1 - kp*y2,
                    dy3dt = ~ kp*RRF*y2),
                ObsEq=list(
                   EX   = ~ 0,
                   OS   = ~ y2,
                   PA   = ~ y3),
States=c("y1","y2","y3"),
Parms=c("ker","RLR","kp","RRF"),
Init=list(0,0,0))

ReservingModel <- nlmeODE(DEmodel,Data)
```

Model 2

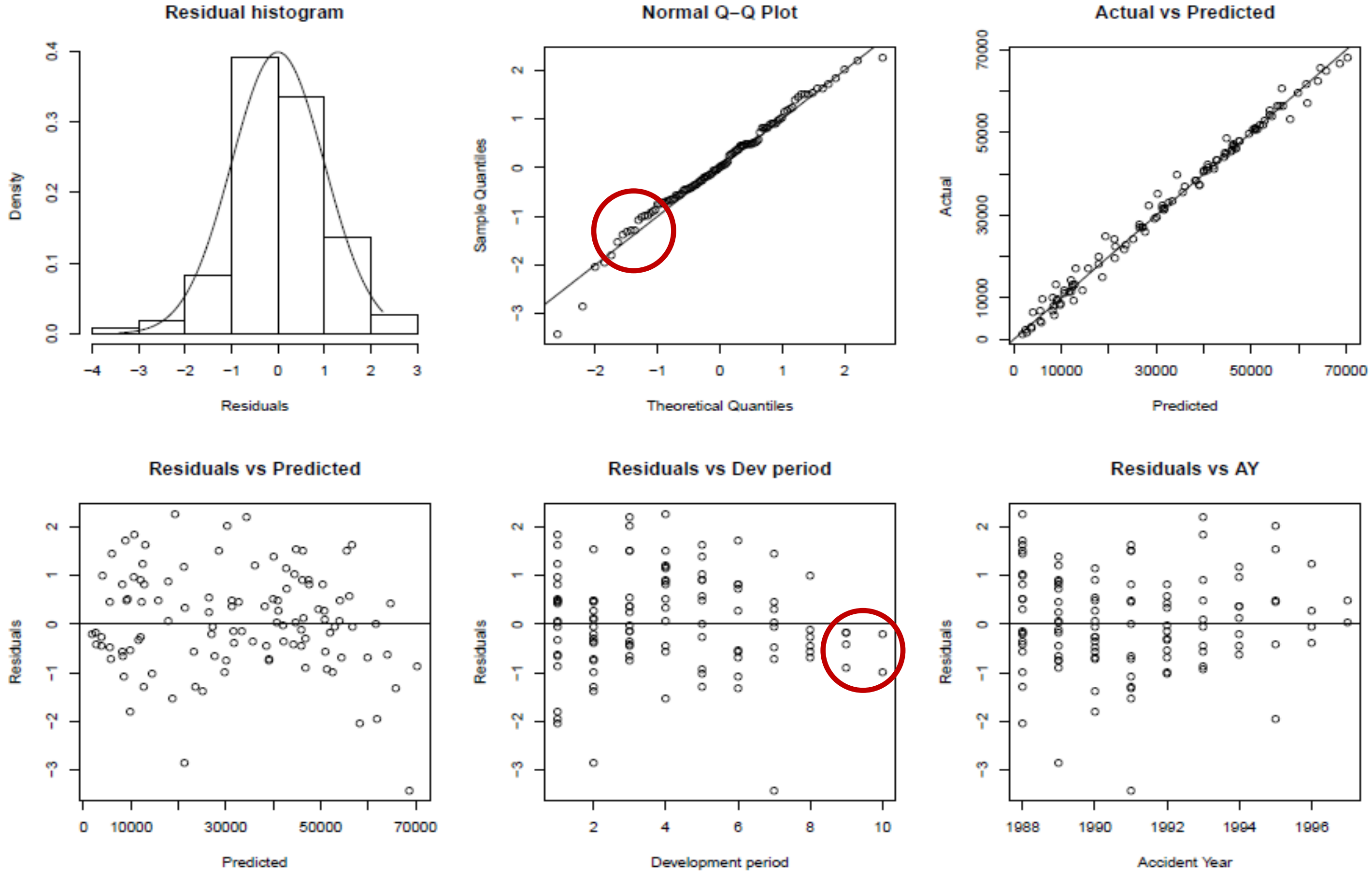


Revise starting values and re-fit nlme model…

Convergence time: 5.7 seconds

Implementation in R

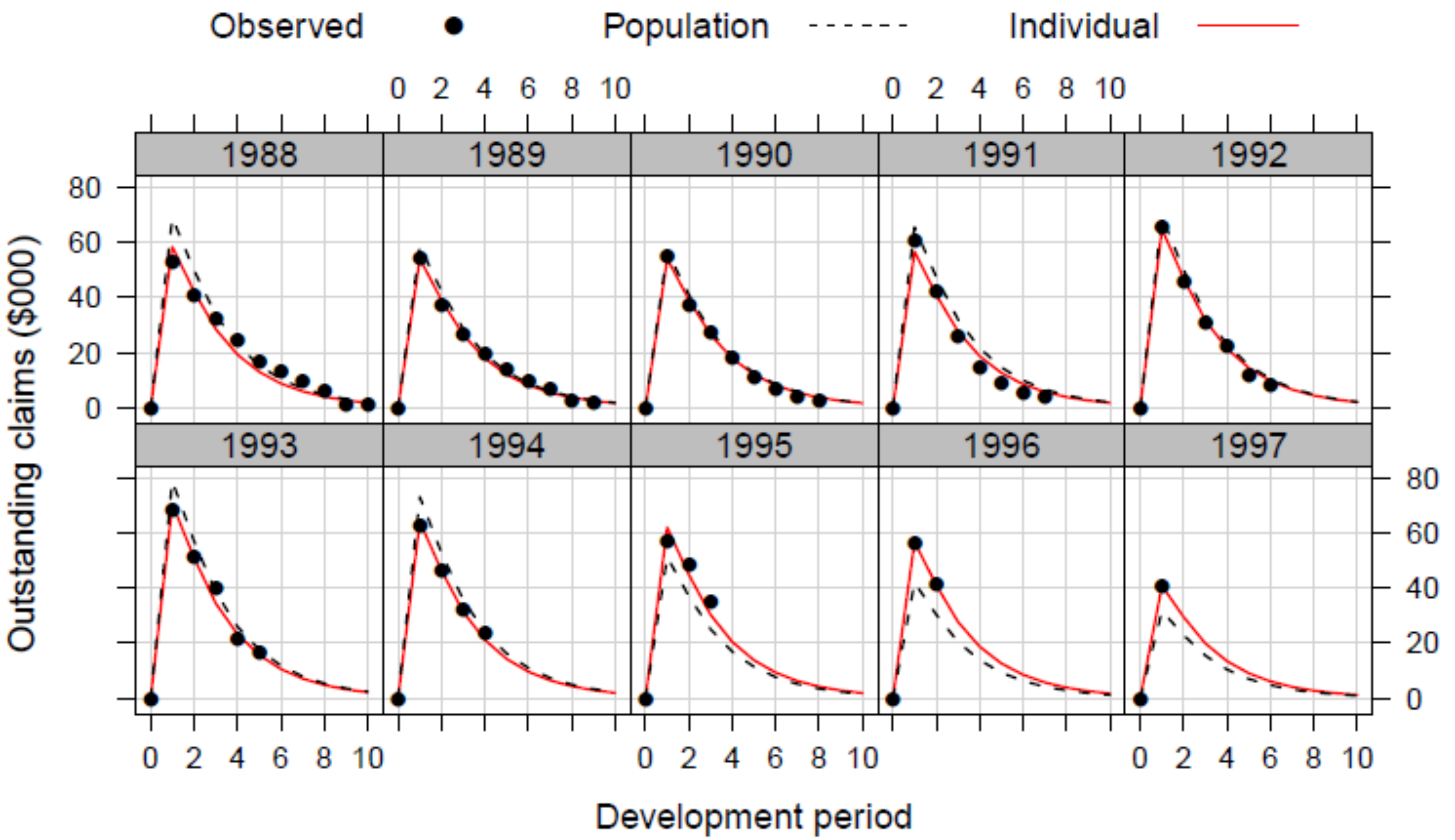
Model 2 Diagnostics



BIC is lower than Model 1

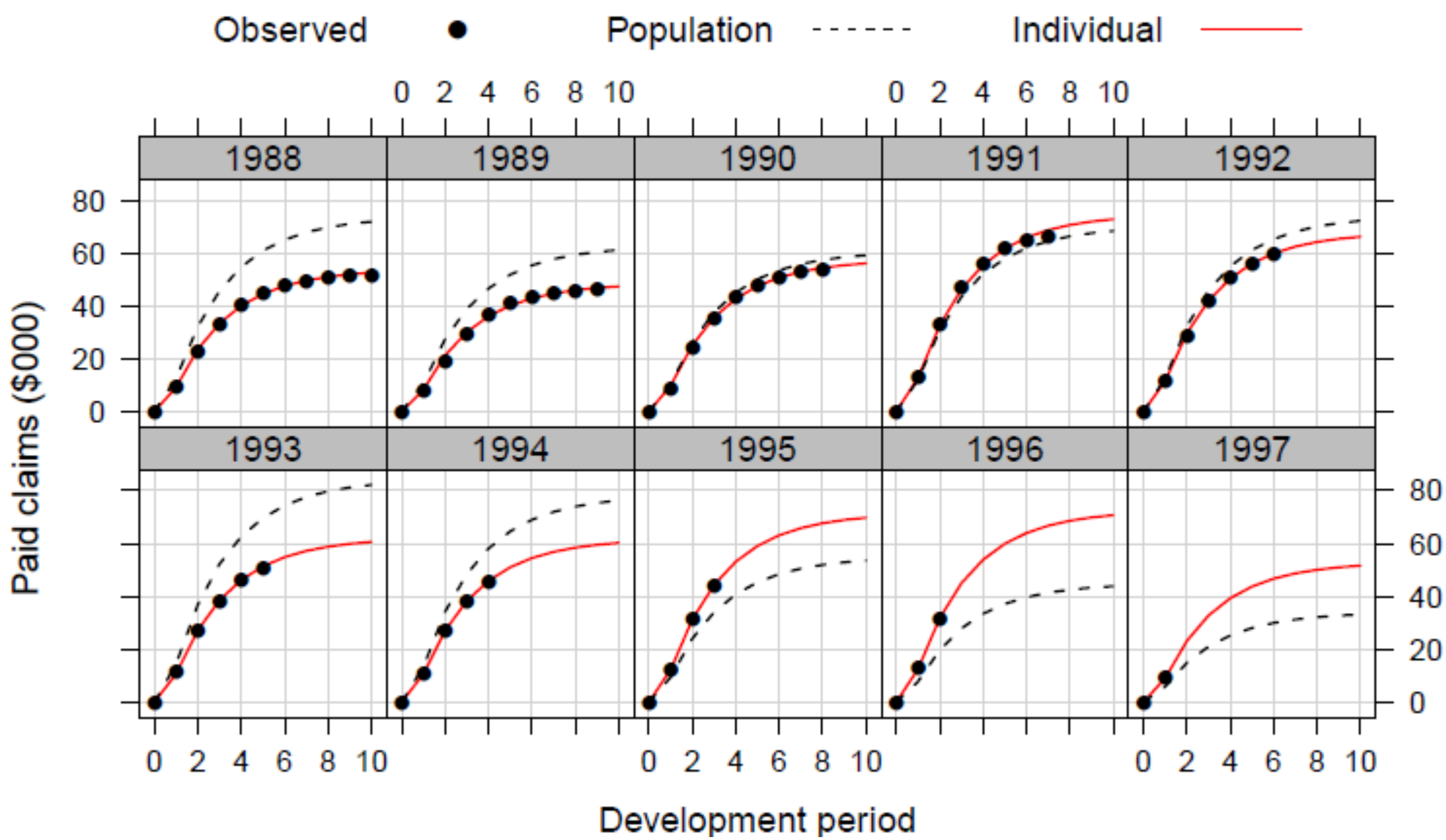
Implementation in R

Model 2 O/S fits



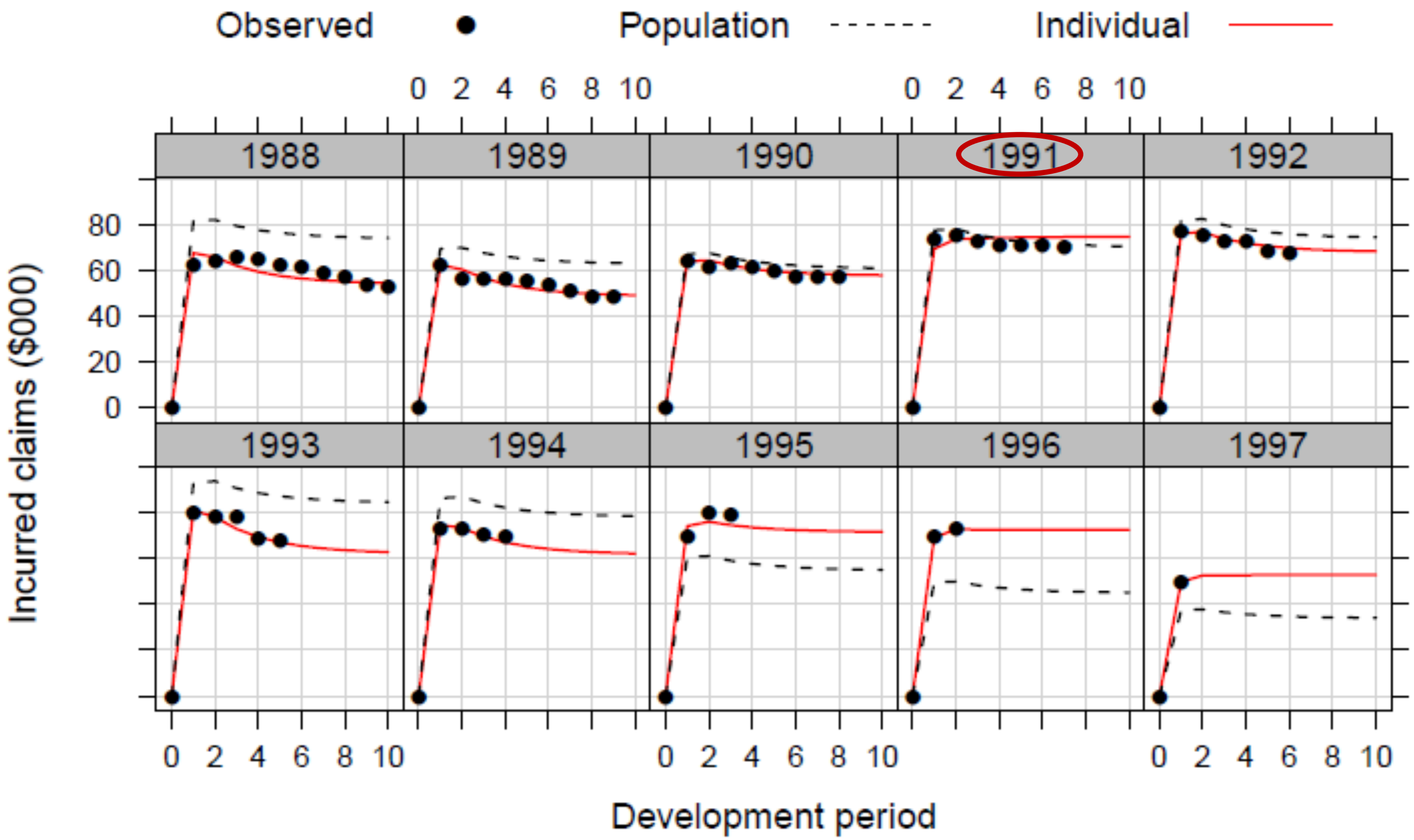
Implementation in R

Model 2 paid fits



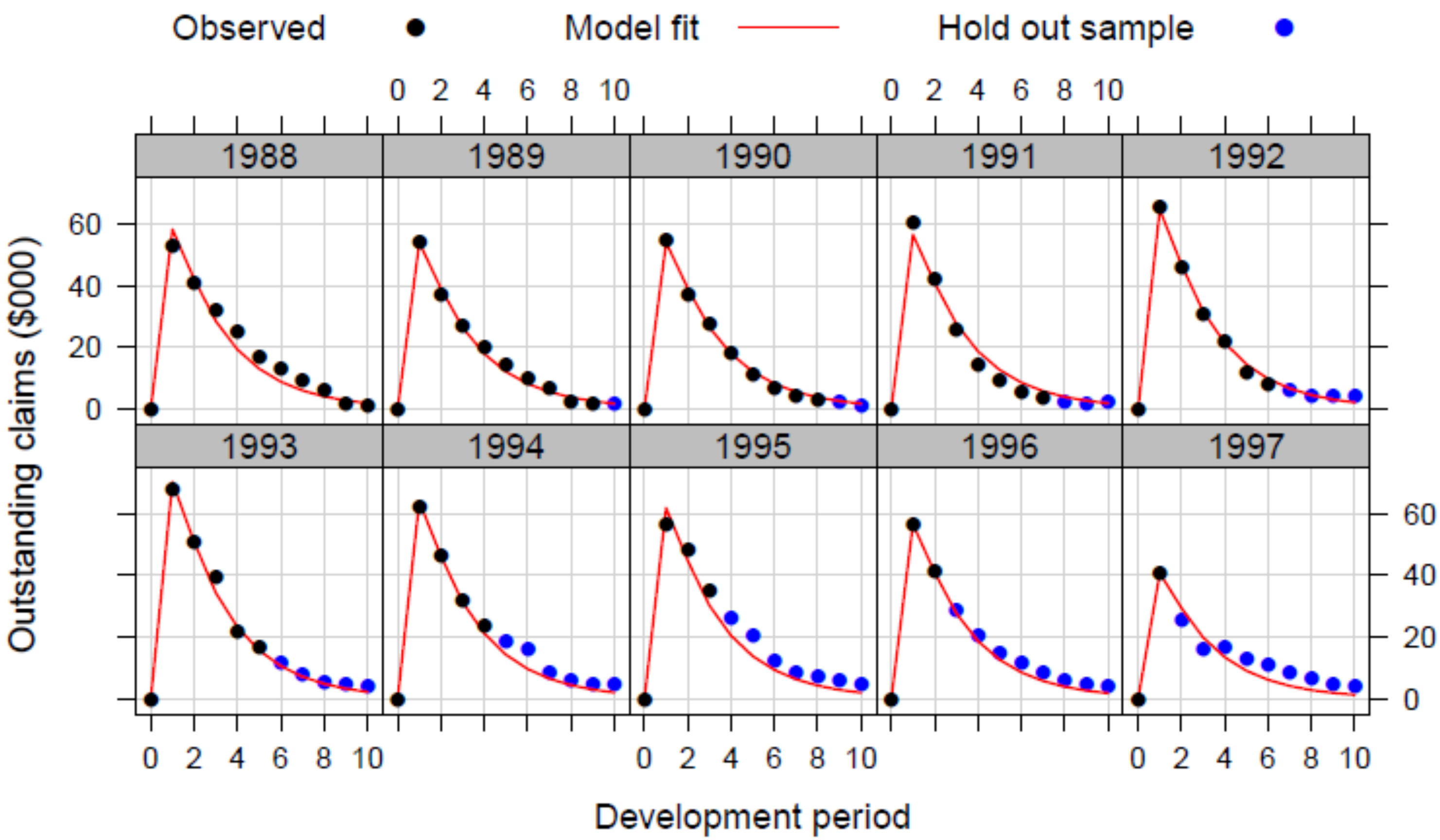
Implementation in R

Model 2 incurred fits



Implementation in R

Model 2 O/S vs. hold out sample



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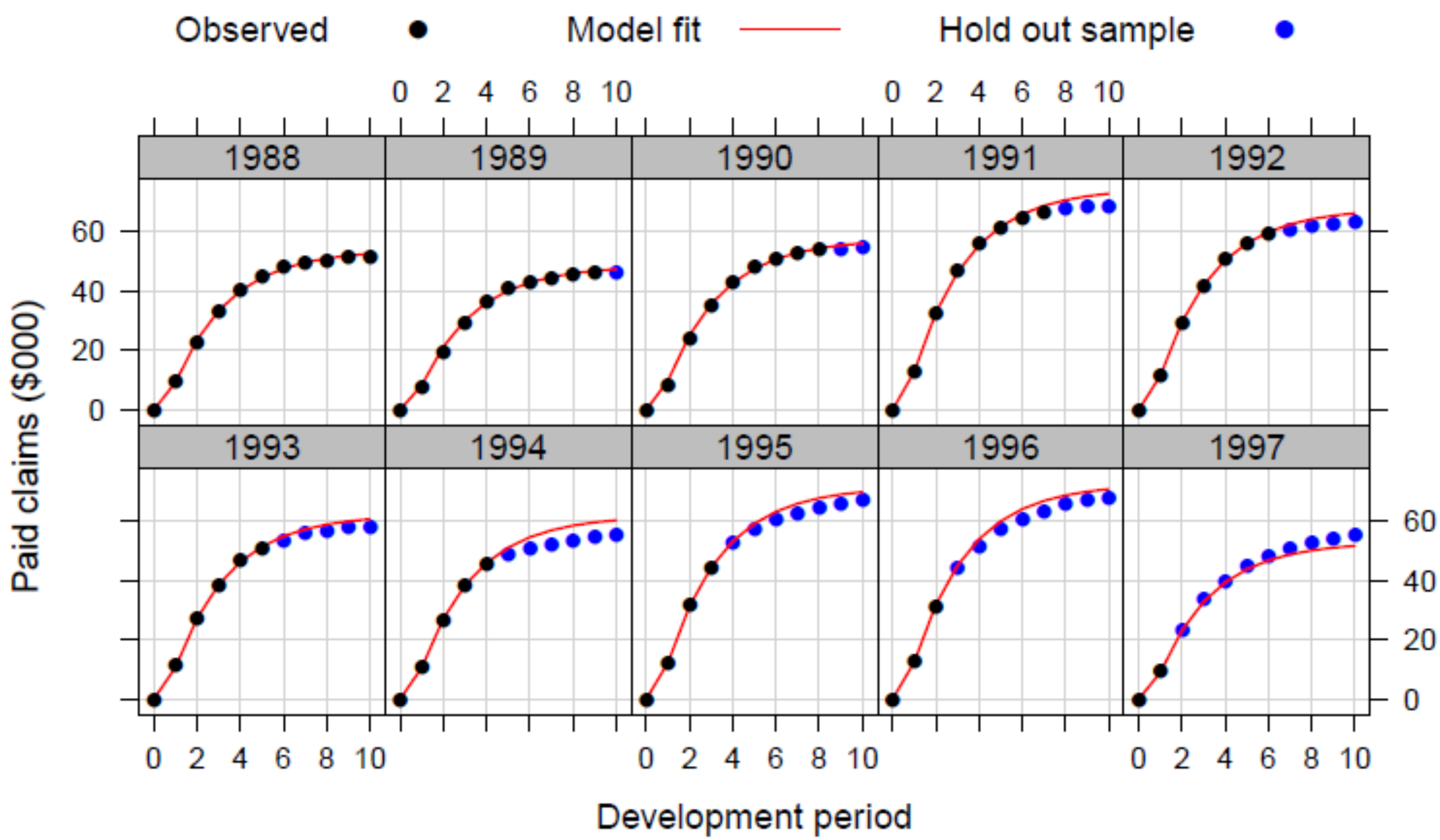
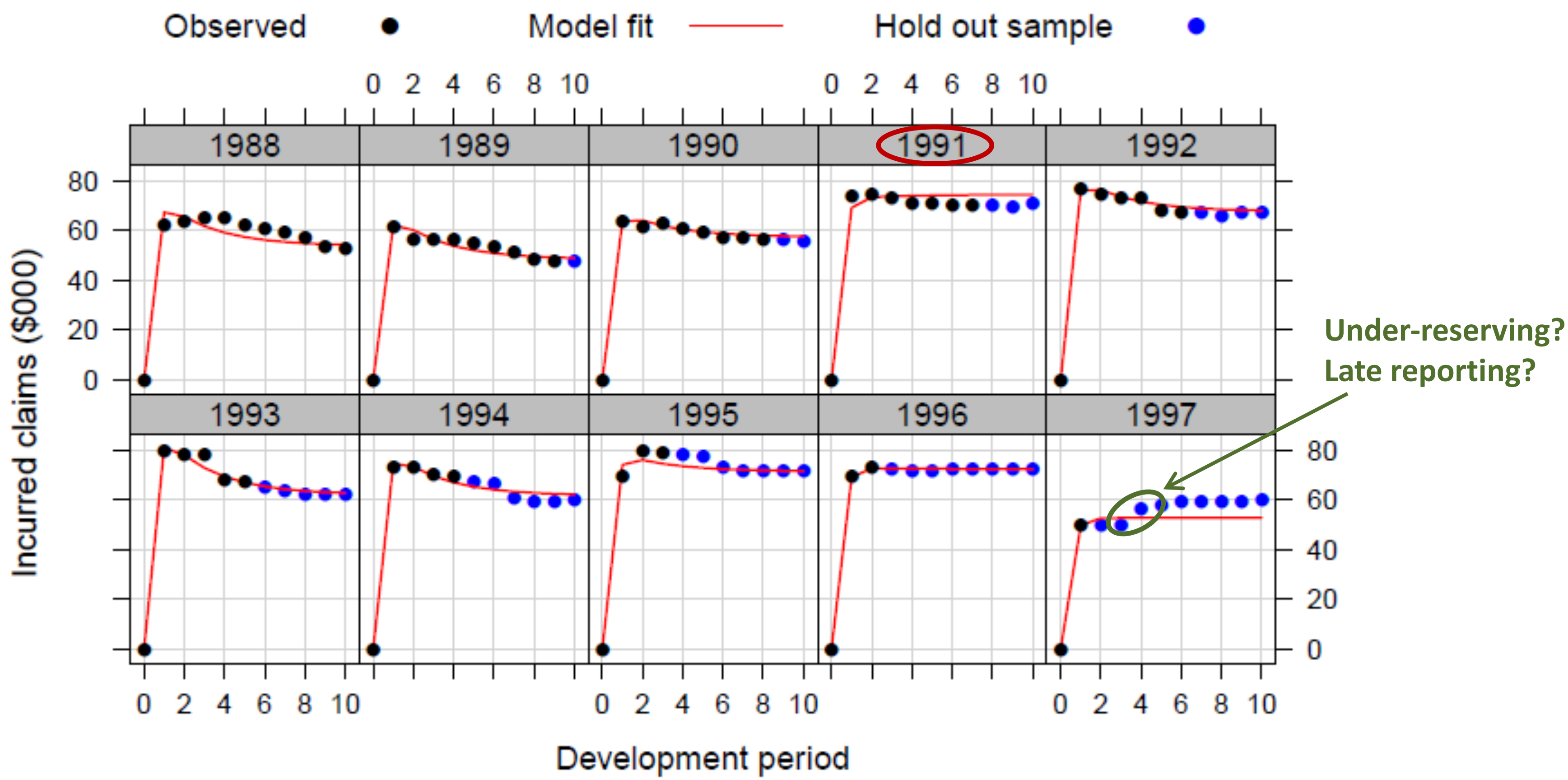
Implementation in R

Model 2 paid vs. hold out sample



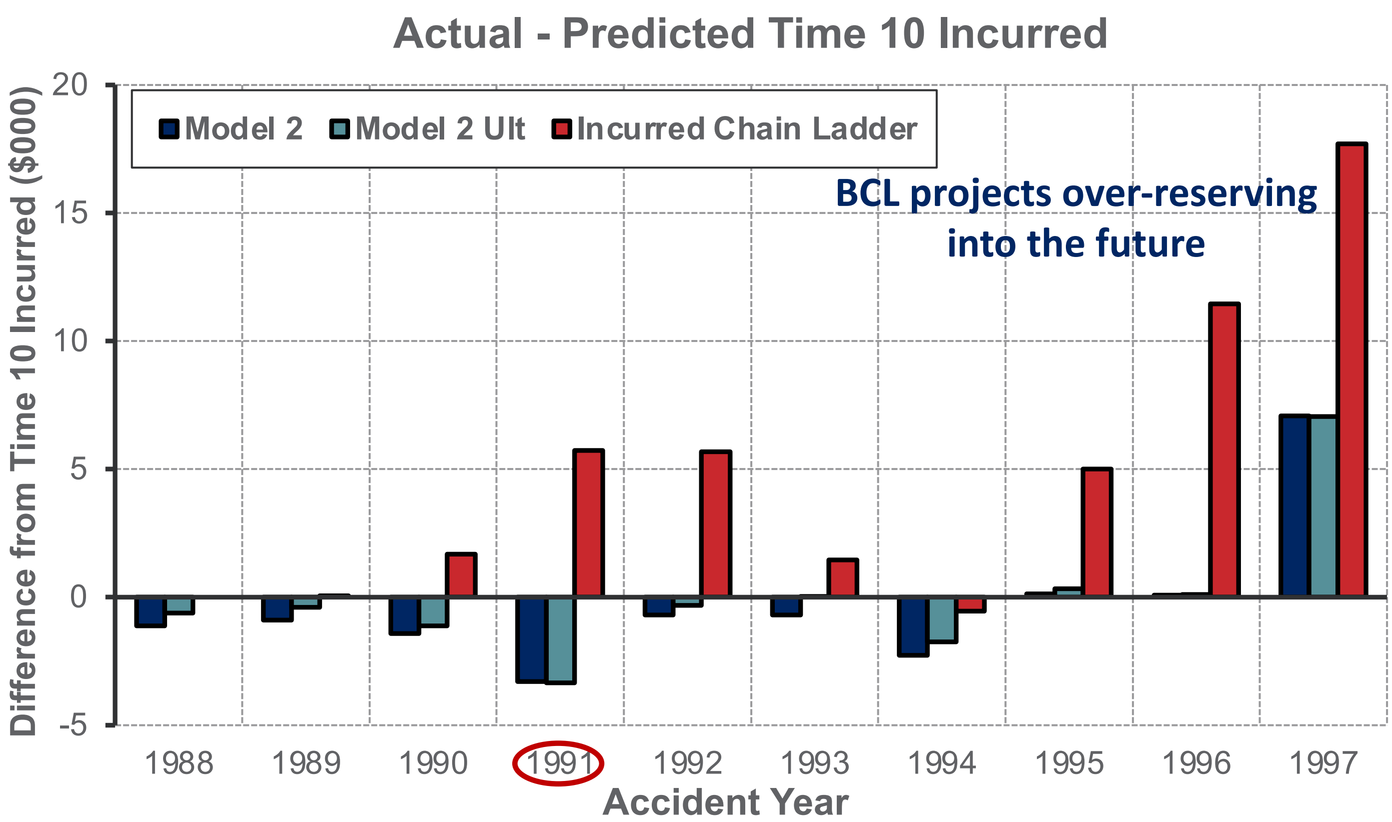
Implementation in R

Model 2 incurred vs. hold out sample



Implementation in R

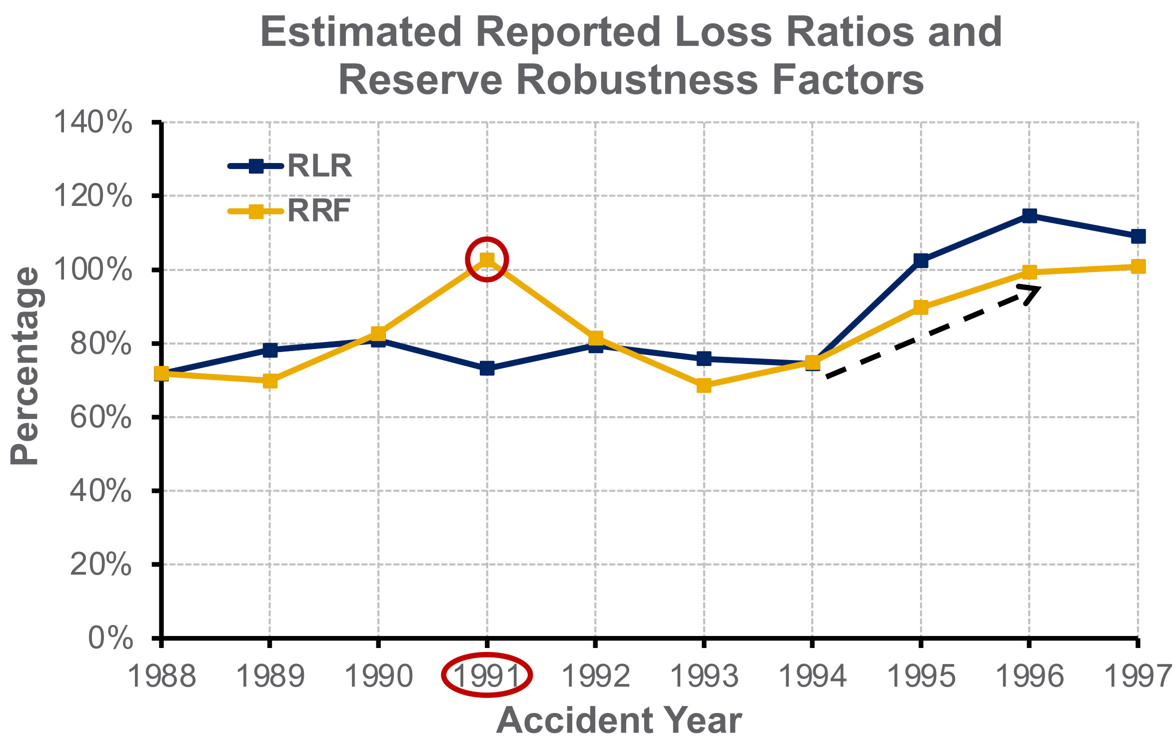
Model 2 Summary (1)



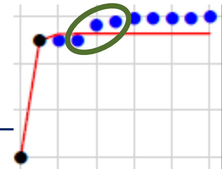
$Ult_i = \mathbf{RLR_i * RRF_i * Prem_i} = Paid_i(t=\infty)$

Implementation in R

Model 2 Summary (2)



Model estimates less over reserving over time…
…but note under reserving in 1997!*

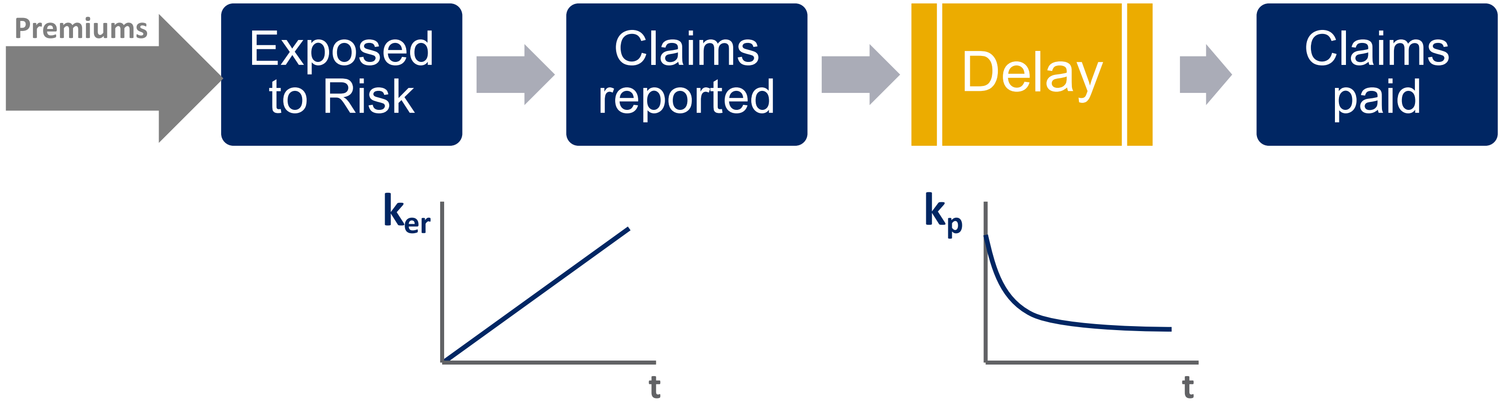


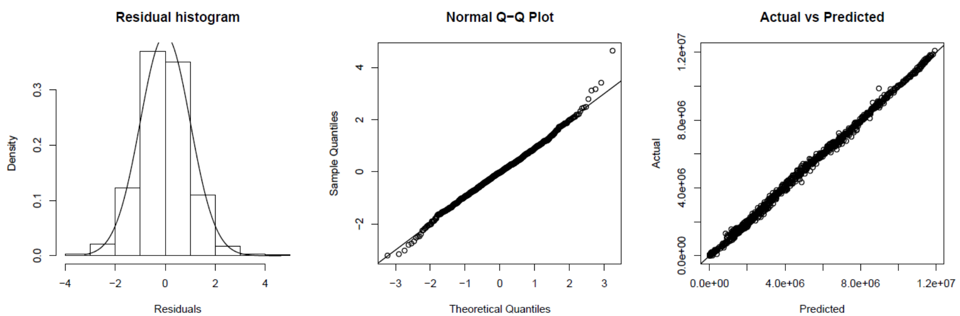
***In practice:** discuss with case handlers and test changes in RRF

Implementation in R

Increase complexity as necessary

Motor PI (capped)* model





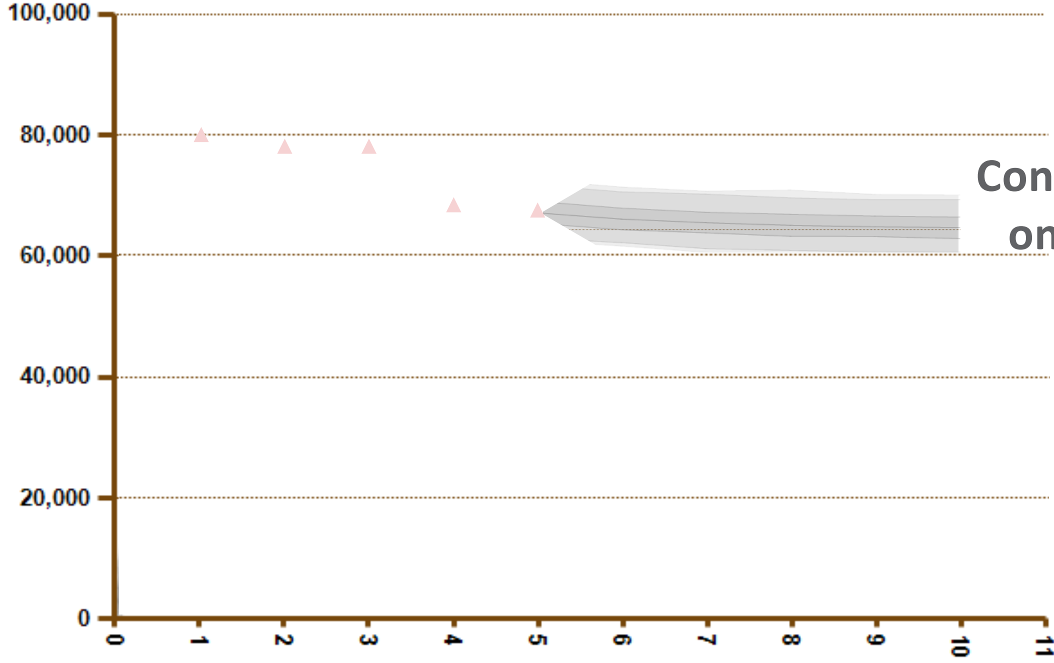
Avg. RRF = 134%

*Personal Injury; claims capped at £100k

Implementation in R

Prediction Intervals

Use model distributional assumptions **OR bootstrap**



Condition simulations on latest incurred?

ror

error

ror

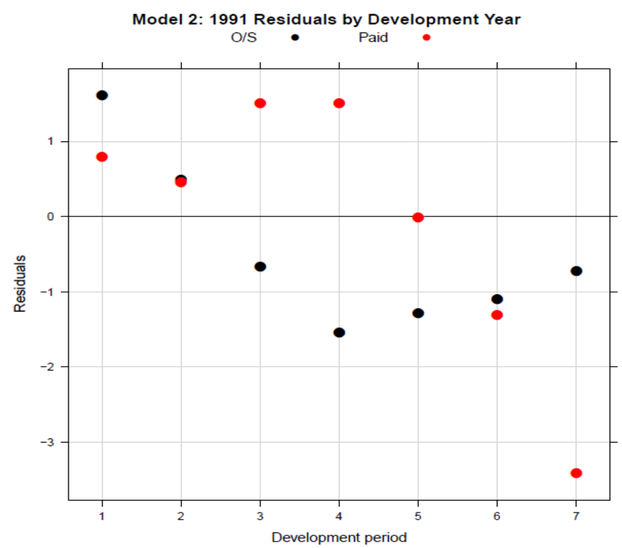
ISSUE – bands are independent of claims history*

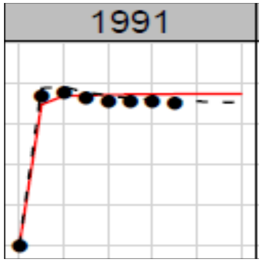
*Intervals show how the entirety of a cohort may have developed if it occurred thousands of times

Future development

Autocorrelation

- **"Repeated measures" models often exhibit autocorrelation**
  - An initial discrepancy in fit is likely to lead to subsequent discrepancy and so on…
  - This can be an issue:





The "nlme" package contains a variety of correlation structures*

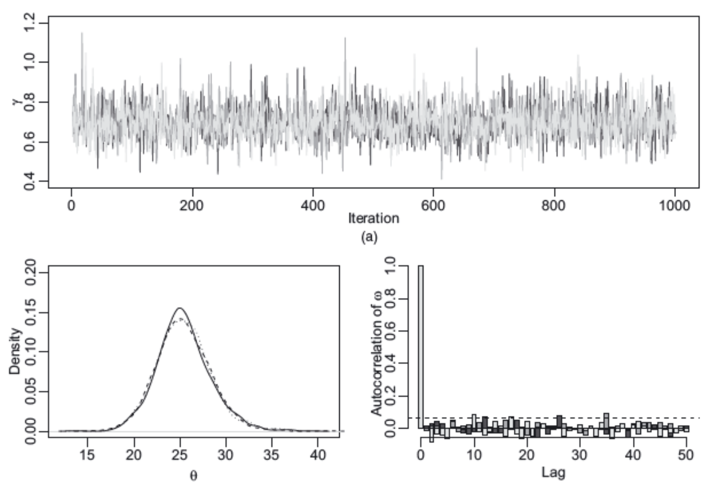
*& user is able to define custom structures

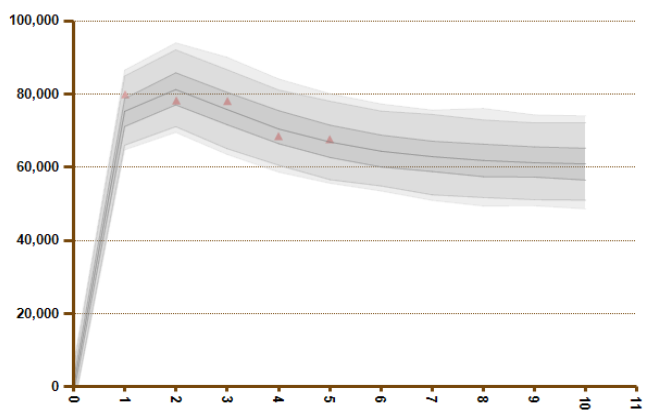
Future development

Bayesian extension

- **A Bayesian extension is a natural next step**
  - A practitioner should be able to select prior distributions for RLR and RRF
  - Potentially not as easy for k_{er} and k_p

- **Key benefit:** obtain a full posterior distribution of outcomes





Conclusions (1)

*Notes from an experienced modeller**

- **"Fitting non-linear mixed effects models can be a tricky (and frustrating) business"**
  - Parameterisation is a key issue
  - If the model runs, it doesn't mean that the answer is correct
  - Method of fitting does make a difference

- **"Model diagnostics are (even more) important for these models"**

"There are some ***general*** rules for fitting these models…

…but experience is the best guide"

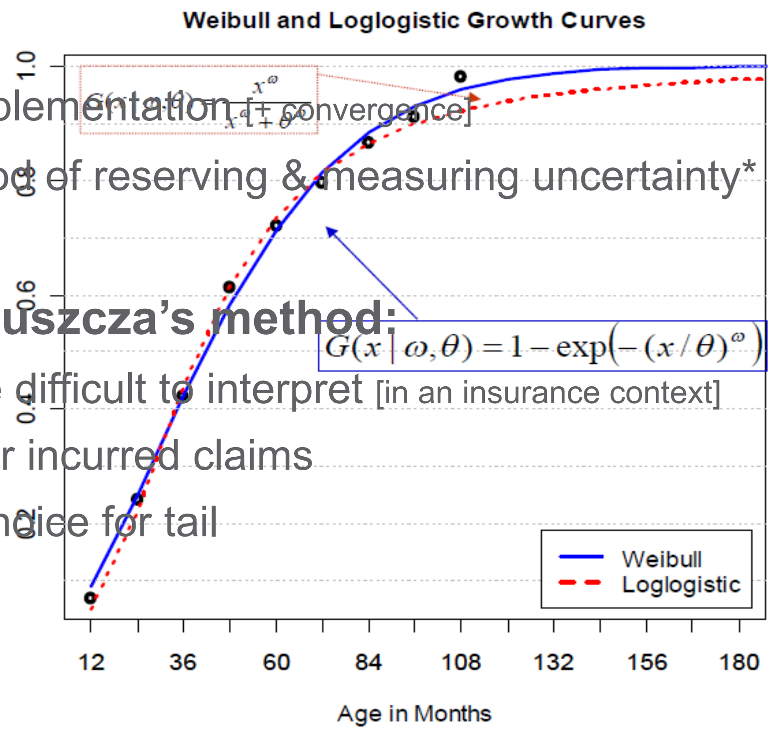
*Mike K Smith - Pharmacometrician, Pfizer

Conclusions (2)

Hierarchical growth curves for loss reserving

- **Strengths of Guszcza's method:**
  - Parsimonious
  - Straightforward implementation [+ convergence]
  - Independent method of reserving & measuring uncertainty*

- **Weaknesses of Guszcza's method:**
  - Parameters can be difficult to interpret [in an insurance context]
  - Often unsuitable for incurred claims
  - Subjective curve choice for tail



How do ... **npare?**

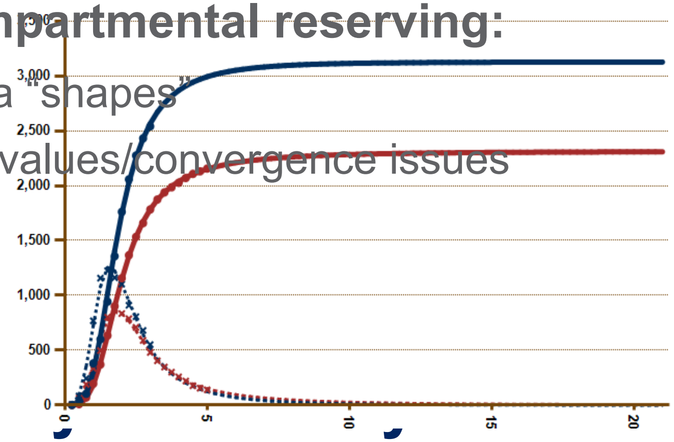
*Zhang, Y., Dukic, V., Guszcza, J. (2012). "A Bayesian Nonlinear Model for Forecasting Insurance Loss Payments"

Conclusions (3)

Compartmental reserving

- **Strengths of compartmental reserving:**
  - Parsimonious yet extensible
  - Interpretable parameters *(including measure of reserve robustness)*
  - Independent method of reserving & measuring uncertainty *(allows negative development)*

- **Weaknesses of compartmental reserving:**
  - Requires specific data "shapes"
  - Sensitivity to starting values/convergence issues
  - Learning curve



*R scripts available on request: Jake.Morris@LibertyGlobalGroup.com

Compartmental Reserving

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Jake Morris

29 June 2015

Liberty Specialty Markets

Additional Slides

Data ⟷ Model

Model form dictates dataset

Cohort	t	Claims	Type	Dose	Cmt
1994	0	0	1	110784	1
1994	0	0	2	0	1
1994	1	62434	1	0	1
1994	1	11194	2	0	1
1994	2	46661	1	0	1
1994	2	26893	2	0	1
1994	3	32248	1	0	1
1994	3	38488	2	0	1
1994	4	24140	1	0	1
1994	4	45580	2	0	1

- **Cohort** cohort
- **t** development period (years)
- **Claims** cumulative O/S or paid at t
- **Type** O/S or paid claims indicator {1,2}
- **Dose** premiums written at t*
- **Cmt** premium input compartment {1}

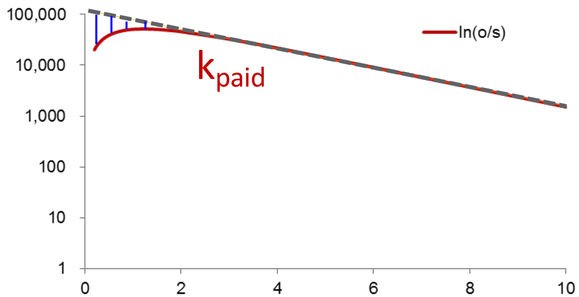
*If using a writing/earnings pattern, "Dose" = premiums written/earned uniformly between t_j and t_{j+1} [Define "Rate" column = Prems ÷ (t_{j+1} - t_j)]

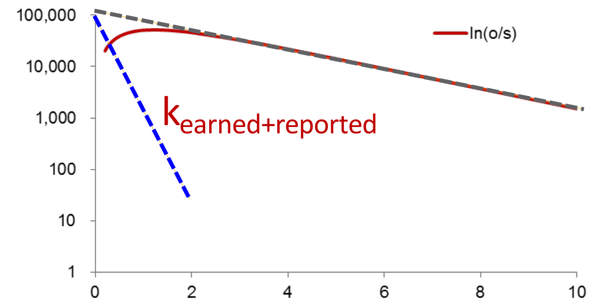
Additional Slides

Parameterisation

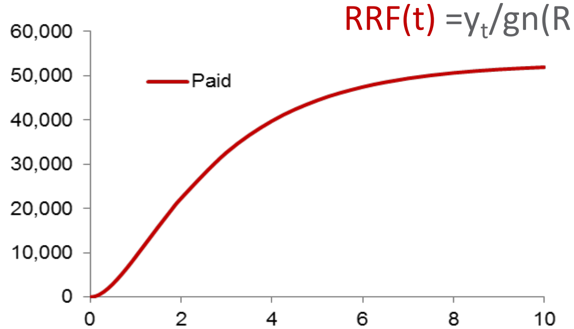
“Feathering” can help

$RLR = fn(Intercept, k_{earned+reported}, k_{paid})$





$RRF(t) = y_t / gn(RLR, k_p, k_{er} | t)$



More complex models require judgement

Methodology

Structural model

Maximum data requirements (e.g. UW year cohorts)

- Cumulative triangles

*Required for UW year data

Liberty Specialty Markets

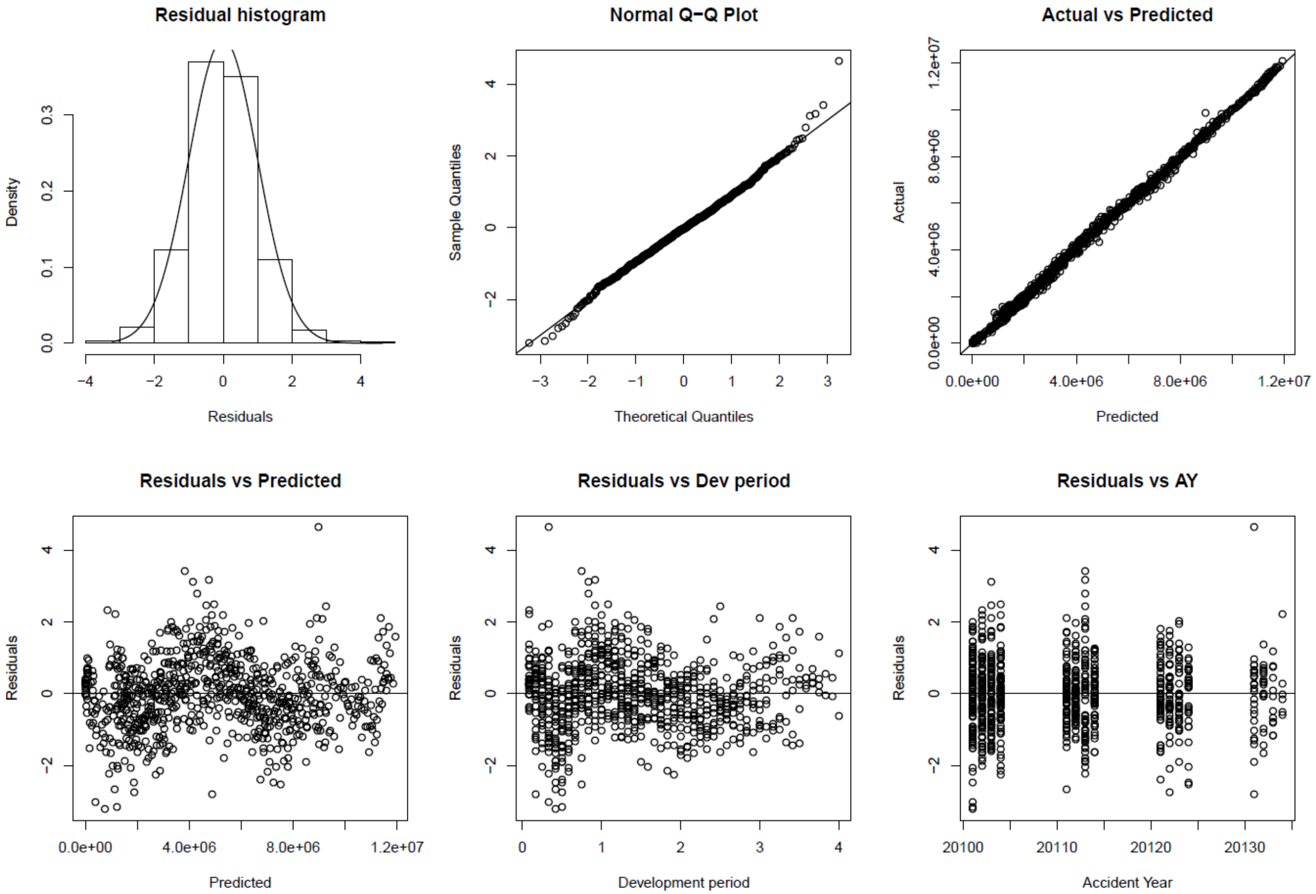
Additional Slides

Wkcomp estimated parameters

AY	RLR	k_r/t	RRF	k_p
1988	0.72	5.94	0.72	0.39
1989	0.78		0.70	
1990	0.81		0.83	
1991	0.73		1.03	
1992	0.79		0.81	
1993	0.76		0.69	
1994	0.74		0.75	
1995	1.02		0.90	
1996	1.15		0.99	
1997	1.09		1.01	

Additional Slides

PI (capped) diagnostics



Additional Slides

Future development: miscellaneous

- **Model extensions**
  - How accurately can we capture the underlying process under this framework?
- **Calendar year effects**
  - Dummy indicators, e.g. RRF(t) = {1,1,1,1,x,x,x}*RRF
  - **Principle:** test significance of adding covariate
- **Predictability study**
  - Compartmental reserving vs. other conventional methods
- **Uncertainty study**
  - Compartmental reserving vs. other conventional methods