Individual reserving – a survey

R in Insurance, Paris 08-June-2017

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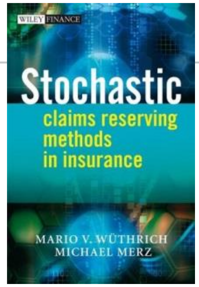
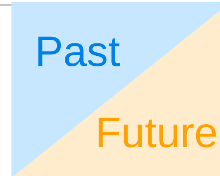
Agenda

1	**Context**
2	Model
3	Results
4	Practical illustration

Context

From aggregate…

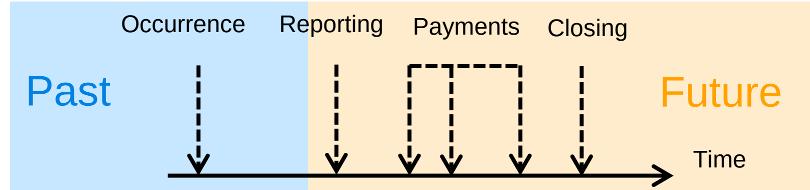




- The current reserving practice consists, in most cases, of using methods based on **claims development triangles** for point estimate projections as well as for capital requirement calculations.
  - The triangles are organised by origin (e.g., accident, underwriting) and development period.
- In the context of an increasing need within the reserving practice for more accurate models, taking advantage of the **information embedded in individual claims data** is a promising alternative compared with the traditional aggregate triangles.
- Traditional reserving methods have worked well in several circumstances in the past
  - Today, however, the awareness of the insurance market about some possible **limitations of traditional aggregate models** to provide robust and realistic estimates in more variable contexts has reached a level which should be noted
- Several potential limits of aggregate models based on triangles have indeed already been highlighted both from a practical and a theoretical point of view:
  - **Over/under-estimation** of the distribution when back-testing realised amounts with forecasts
  - **Huge estimation error** for the latest development periods due to the lack of observed aggregate amounts
  - Uncertainty about the ability of these models to properly capture the **pattern of claim development**, combined with the **limited interpretive and predictive power** of the accident and development period parameters

Context

…to individual-based modelling



- As noted in the report on worldwide non-life reserving practices from the ASTIN Working Party on Non-Life Reserving (June 2016), there is ‘an increase in the need to move towards individual claims reserving and big data, **to better link the reserving process with the pricing process and to be able to better value non-proportional reinsurance**.’

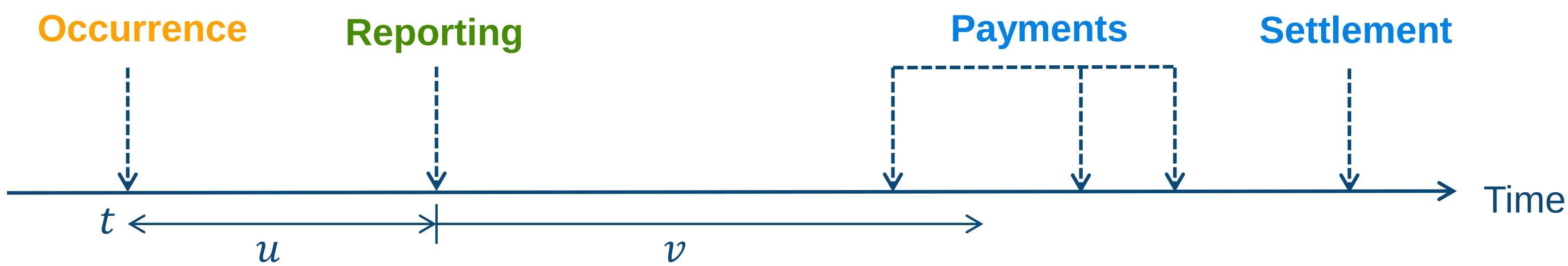
- It is interesting to note that stochastic models for unpaid claims reserving **appeared at around the same time for both individual-based and triangle-based models.**
  - To our knowledge, **Norberg (1983, 1993, 1999), Jewell (1987), Arjas (1989) and Hesselager (1994)** are among the earliest papers which introduced a proper probabilistic setting for individual claims reserving, recently applied by **Antonio and Plat (2014)**
  - To be compared with the stochastic models for triangles in **Mack (1993) and following contributions**
- To date, we suspect that the greater success of the triangle-based models could be driven by their **comparative ease of use (true?)** and the **lack of inexpensive computing power in the early days** of these models.

Agenda

Model

General parametrization of the individual model

- The individual claims paths are modelled with continuous time stochastic processes



Claims occur at times T_n according to some Poisson process with intensity $\lambda(t)$

Claims are **reported** with a delay with distribution $p_{U|t}(du)$

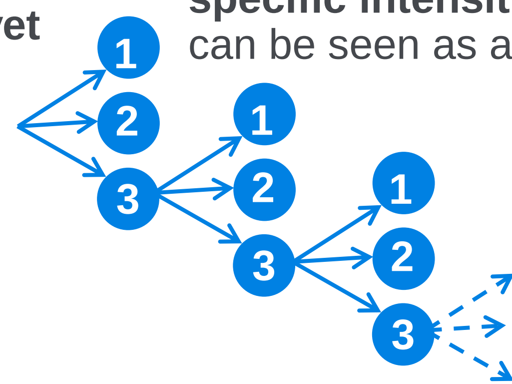
Occurrence and reporting distributions have to be estimated jointly as observation is biased due to hidden **Incurred But Not yet Reported claims (IBNyR)**

Norberg (1983, 1993, 1999)
Hesselager (1994)
Antonio & Plat (2014)

Payments and settlement events are modelled using three types of events:

(1) settlement without payment at settlement
(2) settlement with payment at settlement
(3) payment without settlement

Each type of event (1,2,or 3) occurs according to its **specific intensity parameter** $h_1(v)$, $h_2(v)$ or $h_3(v)$: can be seen as a **recursive competing risks** model



If an event $i \in \{2,3\}$ occurs v time units after reporting, then random payments $Y_i(v)$ are generated

Model
Poisson point measure representation

- **Poisson point measure (PPM):** A powerful tool to study Marked Poisson processes
  - Defined on $\mathbb{R}_+^2$ as $Q(dt, du) = \sum_{n\geq 1} \delta_{(T_n, U_n)}(dt, du)$, it has intensity measure $\lambda(t)\mathrm{dt}\, p_{U|t}(\mathrm{d}u)$
    - Example: $\int_0^\tau \int_0^\infty Q(dt, du) = \sum_{n\geq 1} 1_{T_n \leq \tau}$ is the number of claims which occurred before time τ
  - **Key property 1:** for measurable $A \subset \mathbb{R}_+^2$, $Q(A)$ is a Poisson random variable with parameter
$$\int_A \lambda(t)\mathrm{dt}\, p_{U|t}(\mathrm{d}u)$$
  - **Key property 2:** if $A \cap B = \emptyset$, then the random variables $Q(A)$ and $Q(B)$ are independent
  - **Key property 3:** The events frequency in the set A can be recovered as
$$\int_{u:(t,u)\in A} \lambda(t)\, p_{U|t}(\mathrm{d}u)$$
- **Example:** the number of IBNyR at time τ writes
$$N_\tau{}^{IBNyR} = \int_0^\tau \int_{\tau - t}^\infty Q(\mathrm{d}s, \mathrm{d}u)$$
- Let us denote by $\boldsymbol{X^{(s)}(t_1, t_2)}$ the total payments for claim occurred at time s between t_1 and t_2 time units after occurrence
  - **Example** : the IBNyR future payments write $X_\tau{}^{IBNyR} = \int_0^\tau \int_{\tau - t}^\infty X^{(s)}(0, \infty)\, Q(\mathrm{d}s, \mathrm{d}u)$

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Results

Micro-macro consistency

- What is the **aggregate dynamics** resulting from the micro model ?
- Let us introduce the incremental number triangle as

$$X_{i,j} = \sum_{n\geq 1} 1_{T_n \in [i,i+1)} 1_{T_n + U_n \in [j,j+1)} = \int_i^{i+1} \int_{j-t}^{j+1-t} Q(dt, du)$$

- **Key property 1** shows that $X_{i,j}$ is Poisson distributed with parameter

$$\int_i^{i+1} \int_{j-t}^{j+1-t} \lambda(t) \mathrm{dt}\, p_{U|t}(\mathrm{d}u)$$

This shows that the related triangle is governed by the Poisson model

- **Key property 2** shows that the $(X_{i,j})$ are independent
- **Key property 3** shows that the occurrence intensity of **reported claims** is

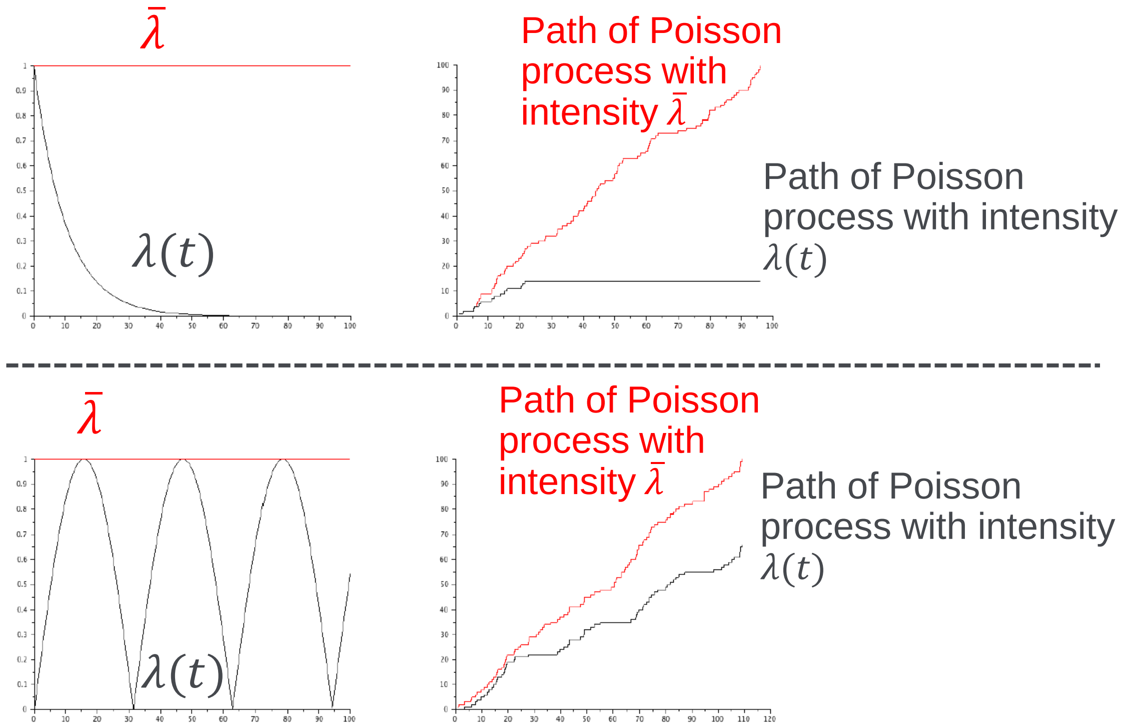
Useful to derive the likelihood

$$t \mapsto \lambda(t) p_{U|t}([0, \tau - t])$$

Results

Simulation of the claims population

- The thinning procedure is a powerful simulation tool to draw future claims paths with general time-dependent frequency parameters
  - Example below : simulation of a non-homogeneous Poisson process



- The intensity $\lambda(t)$ being given, one has to simulate a Poisson process with such intensity
- **Thinning procedure:** assume that this intensity is bounded, that is $\lambda(t) \leq \bar{\lambda}$

-> One is able to easily simulate a Poisson process with intensity $\bar{\lambda}$ as a sequence of $(\bar{T}_n)_{n\geq 1}$ such that the $(\bar{T}_n - \bar{T}_{n-1})$ are iid exponentially distributed with parameter $\bar{\lambda}$

-> Then, select each occurrence $\bar{T}_n$ with probability $\lambda(\bar{T}_n)/\bar{\lambda}$

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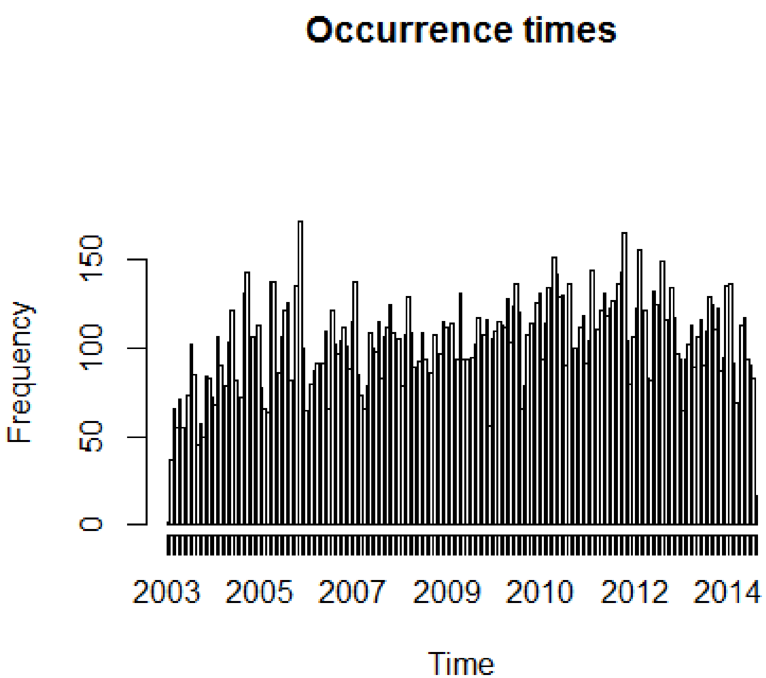
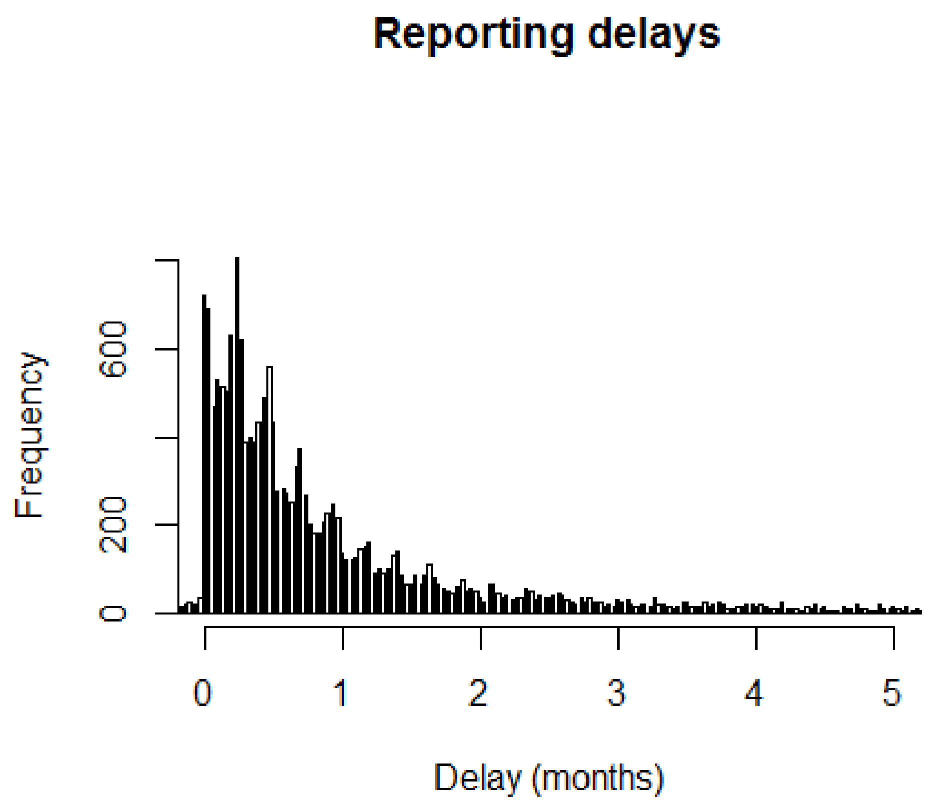
Practical illustration

Data set

R illustration





R illustration

Practical illustration

Forecasting the IBNyR: micro vs macro

	Expected IBNyR	Process error	Estimation error	Prediction error
Mack Chain-Ladder model	328	32,5	107,4	112,3
Individual claims model	217	14,7	2,2	14,9

Reduction in process error: the individual model takes advantage of its Poisson macro-consistency

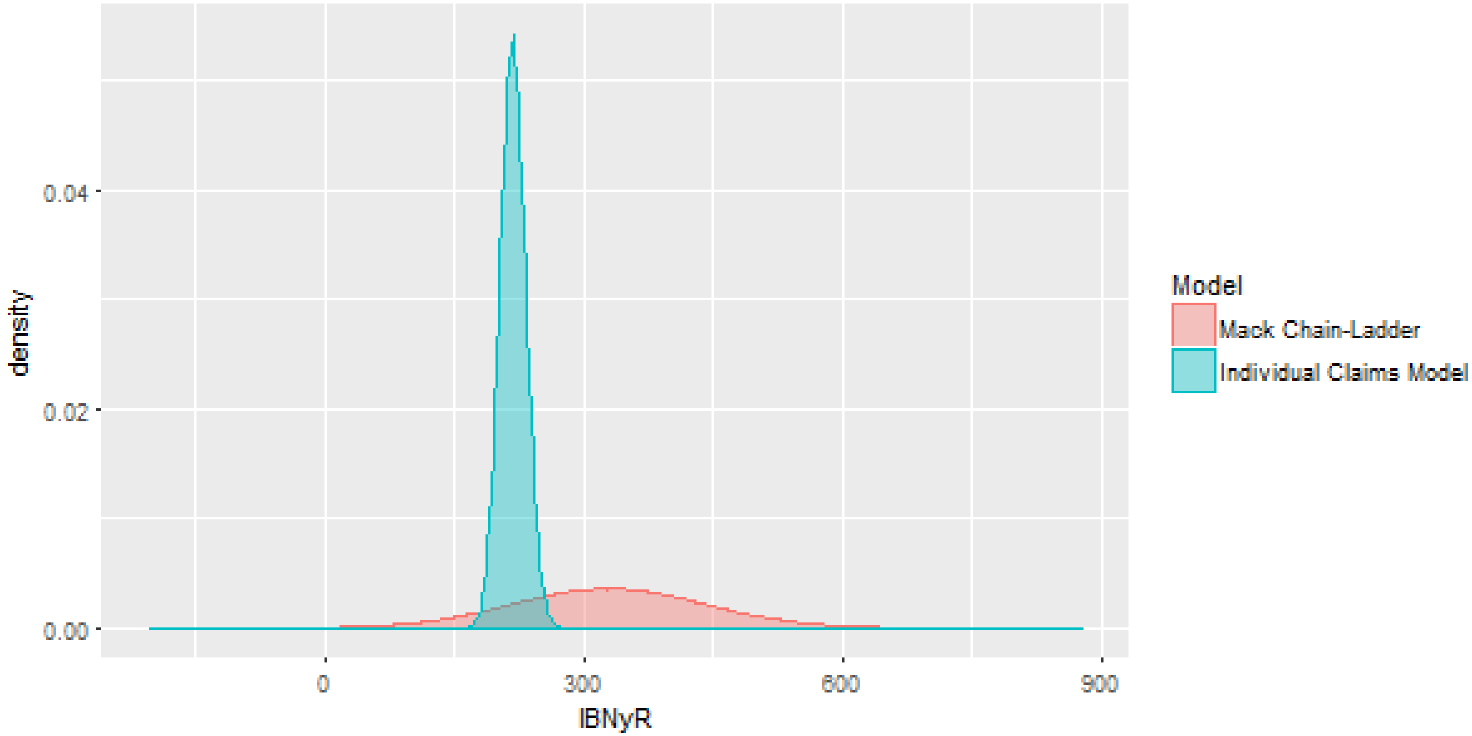
Reduction in estimation error: the individual model takes advantage of the large amount of individual data

Overall reduction in prediction error: the use of the individual model reduces reserves uncertainty

Practical illustration

Forecasting the IBNyR: micro vs macro

R illustration



Use of Mack Chain Ladder of the 'ChainLadder' Package

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Thank you

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